EC307: Mobile Communication and Networks

Module 4: Cellular Traffic Calculation

In cellular communication systems, the study of **traffic calculation** is essential for efficient network planning and resource allocation. It helps in determining how many channels are required in each cell to handle a given number of subscribers while maintaining an acceptable **Grade of Service (GoS)**.

Traffic analysis provides quantitative methods to:

- Estimate the number of channels required per cell.
- Determine blocking probability and system utilization.
- Predict system behavior under different user loads.

1 Basic Concepts

(a) Call and Holding Time

A **call** is a connection request by a subscriber that occupies a radio channel for a certain period called the **holding time** (h). Holding time is usually expressed in seconds or minutes.

(b) Call Arrival Rate

The **call arrival rate** (λ) denotes the number of call attempts per unit time (usually calls per hour). It is generally assumed to follow a **Poisson distribution**.

(c) Traffic Intensity (A)

The average load on a system is called the **traffic intensity**, measured in **Erlangs (E)**.

$$A = \lambda \times h$$

where,

- A = traffic intensity (Erlangs)
- λ = call arrival rate (calls/hour)
- h = average holding time (hours)

1 Erlang means one channel is occupied continuously for one hour.

(d) Grade of Service (GoS)

The **Grade of Service (GoS)** is the probability that a new call will be blocked or delayed due to lack of available channels:

$$\text{GoS} = \frac{\text{Number of blocked calls}}{\text{Total number of call attempts}}$$

Typical values:

- Urban areas: GoS = 0.02 (2%)
- Rural areas: GoS = 0.05 (5%)

(e) Channel Utilization

The efficiency of channel usage is expressed as:

$$\text{Utilization} = \frac{A_c}{N}$$

where A_c = carried traffic (Erlangs) and N = number of available channels.

2 Traffic Models

Two classical traffic models are used in cellular system design.

(a) Erlang B Formula (Loss System)

The Erlang B formula is used for systems where blocked calls are cleared (no waiting queue). It gives the probability of call blocking as:

$$B(A, N) = \frac{\frac{A^N}{N!}}{\sum_{k=0}^{N} \frac{A^k}{k!}}$$

where

- A = offered traffic (Erlangs)
- N = number of channels
- $B(A, N)$ = blocking probability (GoS)

This formula assumes:

- Poisson call arrivals.
- Exponentially distributed holding times.
- Blocked calls are lost (no retrial).

(b) Erlang C Formula (Delay System)

Used when calls can wait in a queue until a channel is free:

$$C(A,N) = \frac{\dfrac{A^N}{N!(1-\frac{A}{N})}}{\displaystyle\sum_{k=0}^{N-1} \frac{A^k}{k!} + \frac{A^N}{N!(1-\frac{A}{N})}}$$

where $C(A,N)$ is the probability that a call will be delayed.

3 Poisson Call Arrival Modeling

In telecommunication systems, call arrivals are often random and independent. A commonly used model for such arrivals is the **Poisson process**.

Let $N(t)$ denote the number of call arrivals in time interval t. $N(t)$ is said to follow a **Poisson distribution** if

$$P[N(t) = k] = \frac{(\lambda t)^k e^{-\lambda t}}{k!}, \qquad k = 0, 1, 2, \ldots$$

where

$$\lambda = \text{average call arrival rate (calls per minute/hour).}$$

Notes:

- Call arrivals are **independent**.
- Probability of one arrival in a very small interval Δt is $\lambda \Delta t$.
- At most one arrival in Δt.
- Inter-arrival time is exponentially distributed.

Example: A mobile switching center receives on average $\lambda = 10$ calls per minute. Find the probability that in a period of $t = 2$ minutes:

i. exactly 15 calls arrive,

ii. fewer than 5 calls arrive,

iii. more than 20 calls arrive.

Solution

Given:

$$\lambda = 10 \text{ calls/min}, \qquad t = 2 \text{ min}$$

$$\lambda t = 20$$

(i) Probability of exactly 15 calls

$$P[N(2) = 15] = \frac{20^{15} e^{-20}}{15!}$$

(ii) Probability of fewer than 5 calls

$$P[N(2) < 5] = \sum_{k=0}^{4} \frac{20^k e^{-20}}{k!}$$

(iii) Probability of more than 20 calls

$$P[N(2) > 20] = 1 - P[N(2) \leq 20] = 1 - \sum_{k=0}^{20} \frac{20^k e^{-20}}{k!}$$

Interpretation

- High value of λt (such as 20) means call traffic is busy.
- Lower probabilities for small k indicate that very low traffic is unlikely.
- Poisson model is useful for analyzing congestion, blocking probability, and MSC performance.

4 Numerical Examples

Example 1: Calculation of Offered Traffic

A cell has 100 subscribers. Each user makes on average 2 calls per hour and each call lasts for 3 minutes. Find the total offered traffic in Erlangs.

$$A = \lambda \times h = (100 \times 2) \times \frac{3}{60} = 10 \text{ Erlangs}$$

Interpretation: On average, 10 channels are busy continuously.

Example 2: Blocking Probability using Erlang B Table

A cell has 15 channels and receives 10 Erlangs of offered traffic. The blocking probability is found using the Erlang B table:

$$B(10, 15) = 0.040$$

Hence, the Grade of Service = 4%. This means 4 out of every 100 call attempts are blocked.

Example 3: Finding Number of Channels Required

If the desired GoS = 0.02 and offered traffic $A = 10$ Erlangs, find N.

From the Erlang B table:

$$B(10, 17) \approx 0.020$$

Hence, 17 channels are required to maintain 2% blocking probability.

Example 4: Mixed User Scenario

Assume there are two types of users in a cell:

- Type 1: 60 users making 1 call/hour, average call duration = 3 min.
- Type 2: 40 users making 3 calls/hour, average call duration = 2 min.

Compute total offered traffic.

$$A_1 = 60 \times 1 \times \frac{3}{60} = 3 \text{ Erlangs}$$
$$A_2 = 40 \times 3 \times \frac{2}{60} = 4 \text{ Erlangs}$$
$$A_{\text{total}} = A_1 + A_2 = 7 \text{ Erlangs}$$

If 12 channels are available, find GoS: From Erlang B table, $B(7, 12) \approx 0.032$ (3.2% blocking).

Example 5: Queue System (Erlang C)

If 10 Erlangs of offered traffic are served by 12 channels, find the probability that a call must wait.

$$C(10, 12) = \frac{\dfrac{10^{12}}{12!(1 - \frac{10}{12})}}{\sum_{k=0}^{11} \dfrac{10^k}{k!} + \dfrac{10^{12}}{12!(1 - \frac{10}{12})}} = 0.208$$

Hence, approximately 20.8% of calls are delayed.