Formal Language & Automata Theory

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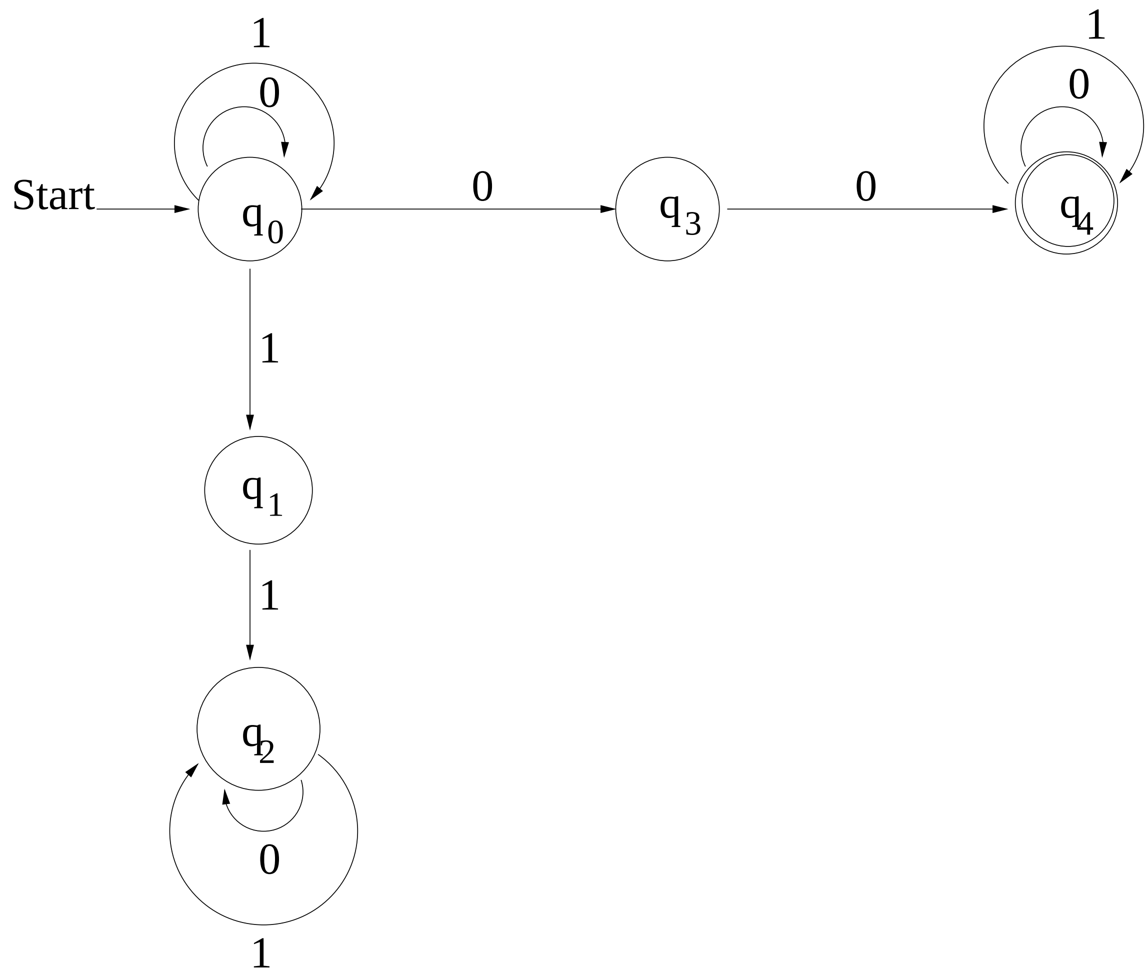
Nondeterministic Finite Automata

- **Nondeterministic**
    - Allow zero, one or more transitions from a state on the same input symbol
- Nondeterministic FA play a central role in
    - The theory of languages and the theory of computation.

Transition Diagram



Nondeterministic Finite Automata

- An input sequence $a_1, a_2, a_3 \ldots a_n$ is accepted by a nondeterministic finite automaton if there exists a sequence of transitions, corresponding to the input sequence, that leads from the initial state to some final state.
  - For example, 01001 is accepted by the NFA shown in the previous slide
- In a DFA, for a given input string w and a state q, there is exactly one path labelled w starting at q. To determine if a string is accepted by a DFA it suffices to check this one path.

Nondeterministic Finite Automata

- For an NFA there may be many paths labeled w, and all must be checked to see whether one or more terminate at a final state.

- When a choice of next state can be made, as in state q_0 on input 0, we may imagine that duplicate copies of the automaton are made.

    - For each possible next state there is one copy of the automaton whose finite control is in that state!

Nondeterministic Finite Automata

- Formally we denote a *nondeterministic finite automaton* by a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where

    - Q set of states
    - Σ input alphabet
    - q_0 start state
    - F set of final states
    - δ is a map from $Q \times \Sigma$ to 2^Q

- The intention of the transition function $\delta(q, a)$ is the **set of all states** p such that there is a transition labelled a from q to p.

Nondeterministic Finite Automata

Q1. Give the function δ for the NFA shown.

Nondeterministic Finite Automata

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The transition function for the given NFA is

	Inputs	
States	0	1
q_0	$\{q_0, q_3\}$	$\{q_0, q_1\}$
q_1	Φ	$\{q_2\}$
q_2	$\{q_2\}$	$\{q_2\}$
q_3	$\{q_4\}$	Φ
q_4	$\{q_4\}$	$\{q_4\}$

Nondeterministic Finite Automata

- To formally describe the behaviour of an FA on a string, we extend the transition function δ to $\hat{\delta}$.

- $\hat{\delta}$ maps $Q \times \Sigma^*$ to 2^Q reflecting sequence of inputs as

1. $\hat{\delta}(q, \epsilon) = \{q\}$
2. $\hat{\delta}(q, wa) = \{p|$ for some state r in $\hat{\delta}(q, w), p$ is in $\delta(r, a)\}$

Condition 2 - Starting at some state q and reading the string w followed by input symbol a, we can be in state p if and only if one possible state we can be in after reading w is r, and from r we may go to p upon reading a.

Nondeterministic Finite Automata

3. $\delta(P, w) = \cup_{q in P} \delta(q, w)$

   Condition 3 - Extension of δ to arguments in $2^Q \times \Sigma^*$, for each set of states $P \subseteq Q$

- The language accepted by a FA designated L(M) where M is the NFA $(Q, \Sigma, \delta, q_0, F)$, is

$$\{w | \delta(q_0, w) \text{ contains a state in } F\}.$$

Nondeterministic Finite Automata

Q4. Given the NFA M and the transition function δ above, is 01001 accepted by the NFA?

$$\delta(q_0, 0) = \{q_0, q_3\}$$

$$\begin{aligned}
\delta(q_0, 01) &= \delta(\delta(q_0, 0), 1) \\
&= \delta(\{q_0, q_3\}, 1) \\
&= \delta(q_0, 1) \cup \delta(q_3, 1) \\
&= \{q_0, q_1\}
\end{aligned}$$

Nondeterministic Finite Automata

$$\begin{aligned}\hat{\delta}(q_0, 010) &= \delta(\hat{\delta}(q_0, 01), 0) \\ &= \delta(\{q_0, q_1\}, 0) \\ &= \delta(q_0, 0) \cup \delta(q_1, 0) \\ &= \{q_0, q_3\}\end{aligned}$$

$$\hat{\delta}(q_0, 0100) = \{q_0, q_3, q_4\}$$

Nondeterministic Finite Automata

$$\begin{aligned}\delta(q_0, 01001) &= \delta(\delta(q_0, 0100), 1) \\ &= \delta(\{q_0, q_3, q_4\}, 1) \\ &= \delta(q_0, 1) \cup \delta(q_3, 1) \cup \delta(q_4, 1) \\ &= \{q_0, q_1, q_4\}\end{aligned}$$

q_4 is in F.

01001 is accepted by the NFA.