Knowledge Representation & Reasoning

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A formalization (Stein)

An <u>inheritance hierarchy</u> $\Gamma = \langle V,E \rangle$ is a directed, acyclic graph (DAG) with positive and negative edges, intended to denote "(normally) is-a" and "(normally) is-not-a", respectively.

- positive edges are written $a \cdot x$
- negative edges are written $a \cdot \neg x$

A sequence of edges is a <u>path</u>:

- a <u>positive path</u> is a sequence of one or more positive edges $a \cdot \ldots \cdot x$
- a <u>negative path</u> is a sequence of positive edges followed by a single negative edge $a \cdot \ldots \cdot v \cdot \neg x$

  Note: there are no paths with more than 1 negative edge.

  Also: there might be 0 positive edges.

A path (or <u>argument</u>) supports a <u>conclusion</u>:

- $a \cdot \ldots \cdot x$ supports the conclusion $a \rightarrow x$ (a is an x)
- $a \cdot \ldots \cdot \neg x$ supports $a \not\rightarrow x$ (a is not an x)

  Note: a conclusion may be supported by many arguments

However: not all arguments are equally believable...

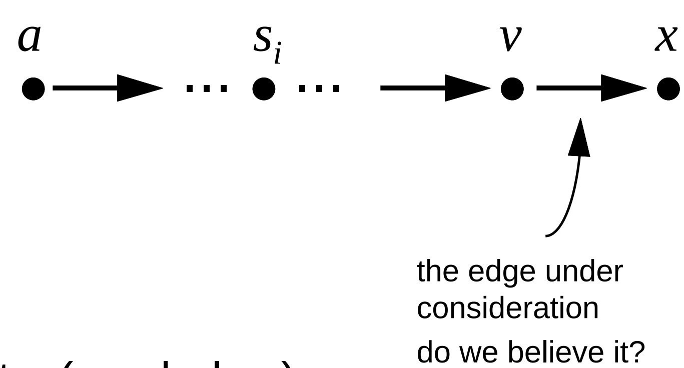
Support and admissibility

Γ supports a path $a \cdot s_1 \cdot \ldots \cdot s_n \cdot (\neg)x$ if the corresponding set of edges $\{a \cdot s_1, \ldots, s_n \cdot (\neg)x\}$ is in E, and the path is <u>admissible</u> according to specificity (see below).

> the hierarchy supports a conclusion $a \to x$ (or $a \not\to x$)
> if it supports some corresponding path

A path is <u>admissible</u> if every edge in it is admissible.

An edge $v \cdot x$ is <u>admissible</u> in Γ wrt a if there is a positive path $a \cdot s_1 \ldots s_n \cdot v$ ($n \geq 0$) in E and



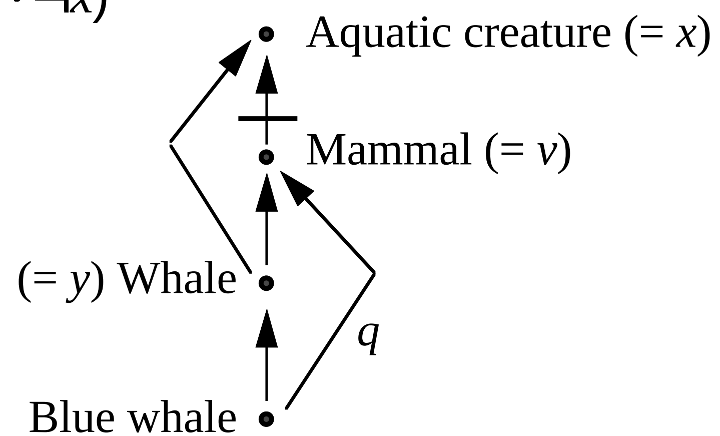
1. each edge in $a \cdot s_1 \ldots s_n \cdot v$ is admissible in Γ wrt a (recursively);
2. no edge in $a \cdot s_1 \ldots s_n \cdot v$ is redundant in Γ wrt a (see below);
3. no intermediate node $a, s_1, \ldots, s_n$ is a preemptor of $v \cdot x$ wrt a (see below).

A negative edge $v \cdot \neg x$ is handled analogously.

Preemption and redundancy

A node y along path $a \cdot ... y ... \cdot v$ is a <u>preemptor</u> of the edge $v \cdot x$ wrt a if $y \cdot\neg x \in E$ (and analogously for $v \cdot \neg x$)

for example, in this figure the node Whale preempts the negative edge from Mammal to Aquatic creature wrt both Whale and Blue whale



A positive edge $b \cdot w$ is <u>redundant</u> in Γ wrt node a if there is some positive path $b \cdot t_1 \ldots t_m \cdot w \in E$ $(m \geq 1)$, for which

1. each edge in $b \cdot t_1 \ldots \cdot t_m$ is admissible in Γ wrt a;
2. there are no c and i such that $c \cdot \neg t_i$ is admissible in Γ wrt a;
3. there is no c such that $c \cdot \neg w$ is admissible in Γ wrt a.

The edge labelled q above is redundant

The definition for a negative edge $b \cdot \neg w$ is analogous

Credulous extensions

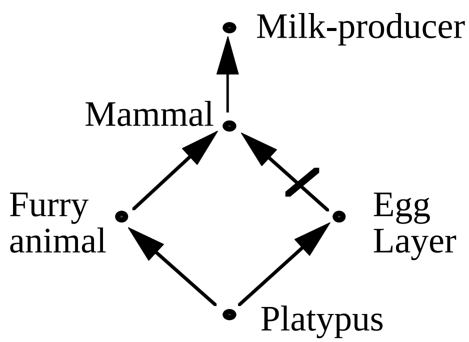
Γ is <u>a-connected</u> iff for every node x in Γ, there is a path from a to x, and for every edge $v \cdot (\neg) x$ in Γ, there is a *positive* path from a to v.

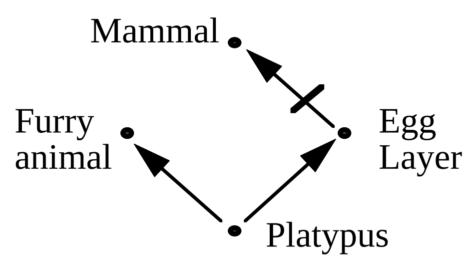
In other words, every node and edge is reachable from a

Γ is (potentially) <u>ambiguous</u> wrt a node a if there is some node $x \in V$ such that both $a \cdot s_1 \ldots s_n \cdot x$ and $a \cdot t_1 \ldots t_m \cdot \neg x$ are paths in Γ

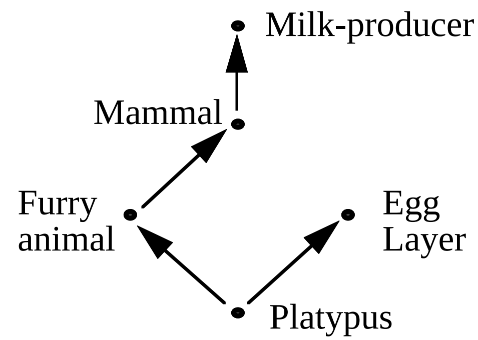
A <u>credulous extension</u> of Γ wrt node a is a maximal unambiguous a-connected subhierarchy of Γ wrt a

If X is a credulous extension of Γ, then adding an edge of Γ to X makes X either ambiguous or not a-connected





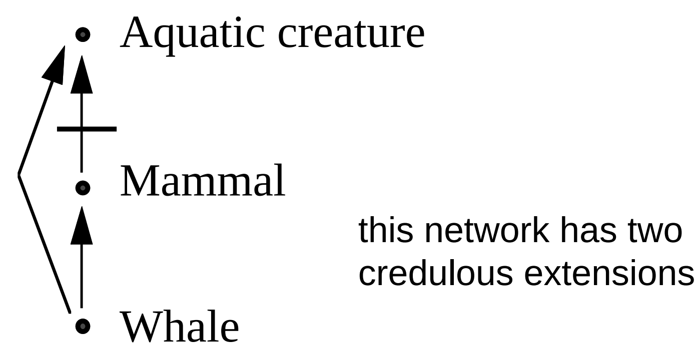
Extension 1



Extension 2

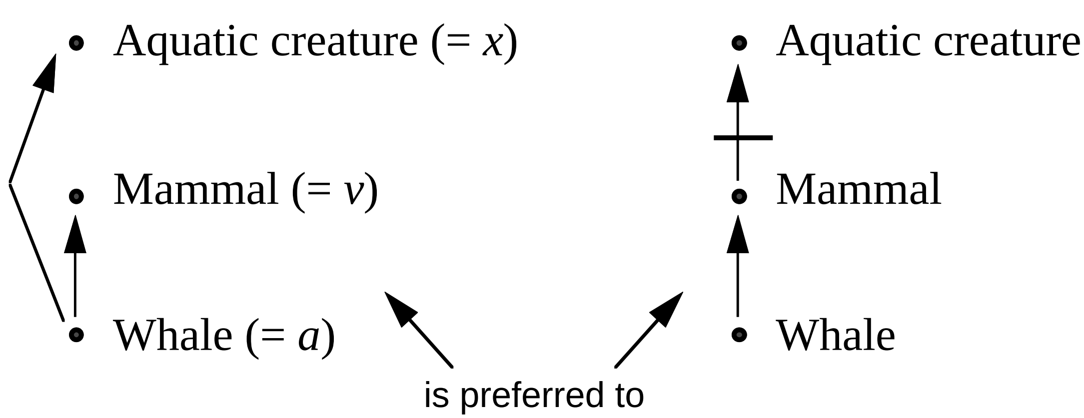
Preferred extensions

Credulous extensions do not incorporate any notion of admissibility or preemption.



Let X and Y be credulous extensions of Γ wrt node a. X is <u>preferred</u> to Y iff there are nodes v and x such that:

- X and Y agree on all edges whose endpoints precede v topologically,
- there is an edge $v \cdot x$ (or $v \cdot \neg x$) that is *inadmissible* in Γ,
- this edge is in Y, but not in X.



A credulous extension is a <u>preferred extension</u> if there is no other extension that is preferred to it.

Subtleties

What to believe?

- “credulous” reasoning: choose a preferred extension and believe all the conclusions supported
- “skeptical” reasoning: believe the conclusions from any path that is supported by all preferred extensions
- “ideally skeptical” reasoning: believe the conclusions that are supported by all preferred extensions

  note: ideally skeptical reasoning cannot be computed in a path-based way (conclusions may be supported by different paths in each extension)

We’ve been doing “upwards” reasoning

- start at a node and see what can be inherited from its ancestor nodes
- there are many variations on this definition; none has emerged as the agreed upon, or “correct” one
- an alternative looks from the top and sees what propagates down

  upwards is more efficient