Computer Graphics: CO 303

Lecture 3

Draconifors

Transformations in 3-D, Rotation and Shear

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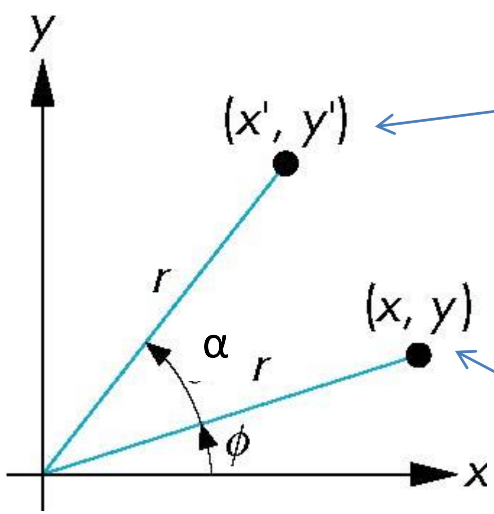
Recap: Most Basic Transformations

- Translation
- Scaling
- Rotation
- Shear

Rotation

- Consider rotation about the origin by Θ degrees
  - Angle increased by Θ, while the radius stays the same.
- Use Polar Coordinates: $(r, \phi) \rightarrow (x, y)$



$x' = r\cos(\phi + \alpha)$

$y' = r\sin(\phi + \alpha)$

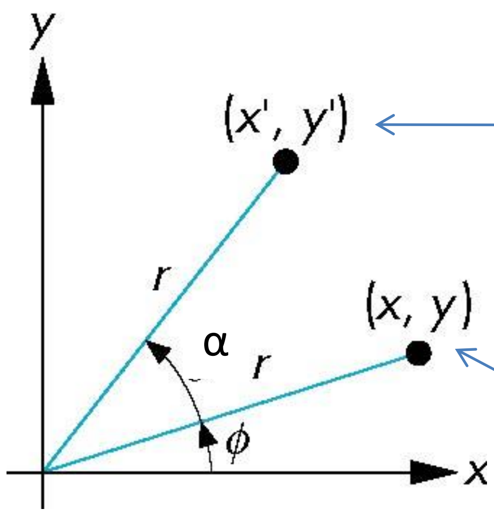
$x = r\cos(\phi)$

$y = r\sin(\phi)$

Rotation

- Consider rotation about the origin by Θ degrees
  - Angle increased by Θ, while the radius stays the same.

$\sin(A + B) = \sin A \cos B + \cos A \sin B$
$\cos(A + B) = \cos A \cos B - \sin A \sin B$

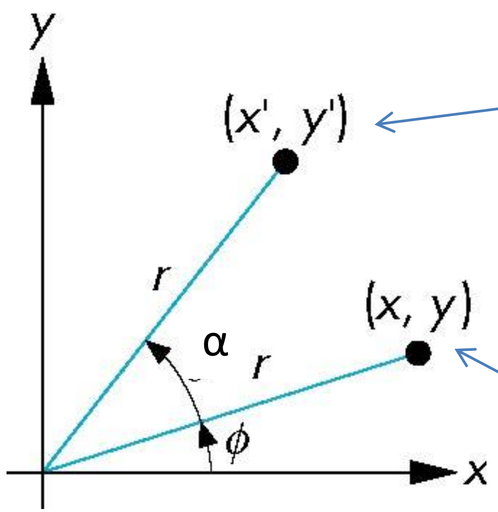


$x' = r \cos(\phi + \alpha)$
$\quad = r(\cos \phi \cos \alpha - \sin\phi \sin \alpha)$
$y' = r \sin(\phi + \Theta)$
$\quad = r(\sin \phi \cos \alpha + \cos \phi \sin \alpha)$

$x = r \cos(\phi)$
$y = r \sin(\phi)$

Rotation

- Consider rotation about the origin by Θ degrees
  - Angle increased by Θ, while the radius stays the same.

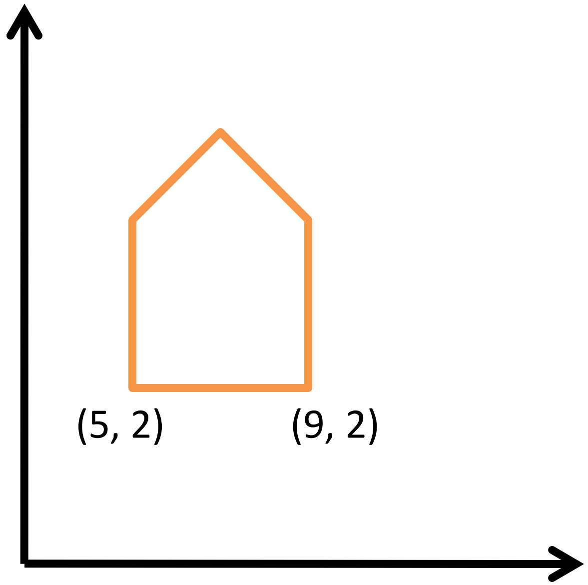


$$x' = x \cos\alpha - y \sin\alpha$$

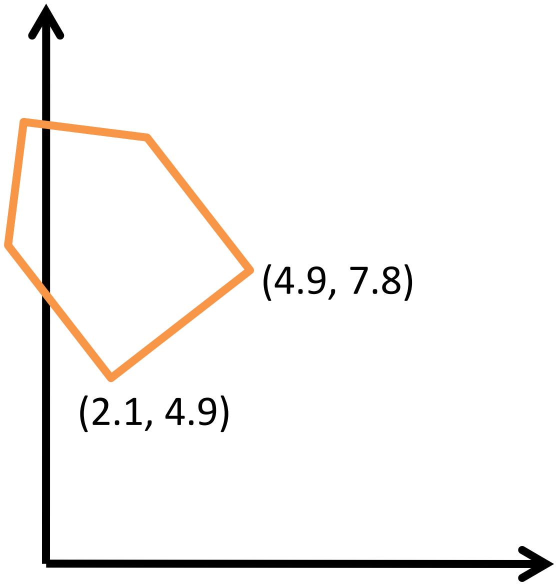
$$y' = x \sin\alpha + y \cos\alpha$$

$$x = r\cos(\phi)$$
$$y = r\sin(\phi)$$

Rotation



Before rotation

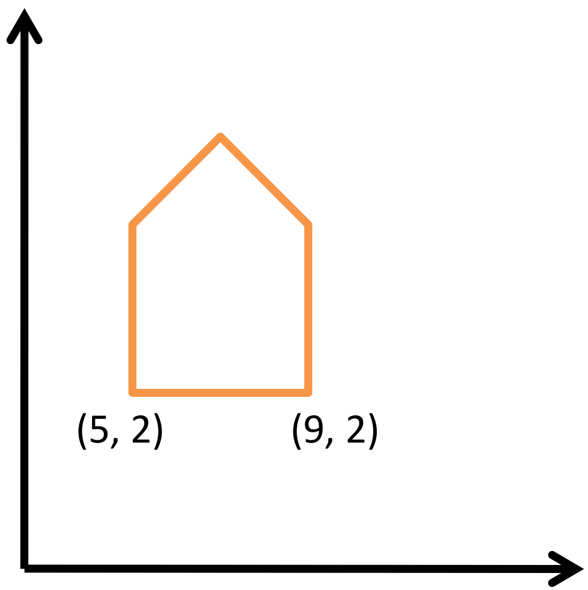


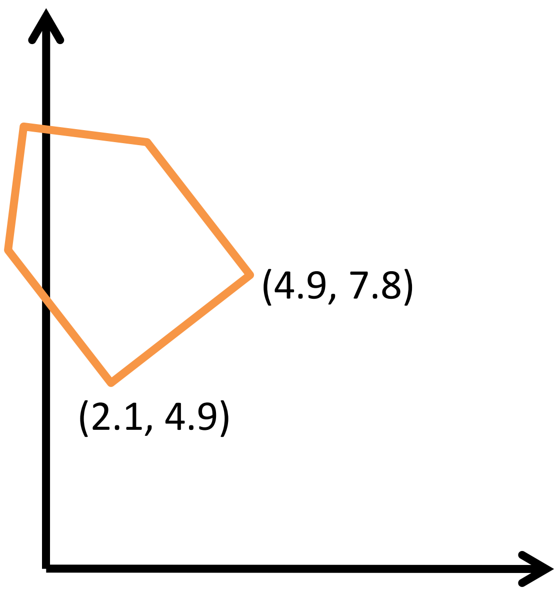
After rotation

- Positive angles: anti clockwise
- Negetive angles: $\cos(-\alpha) = \cos(\alpha)$

  $\sin(-\alpha) = -\sin(\alpha)$

Rotation





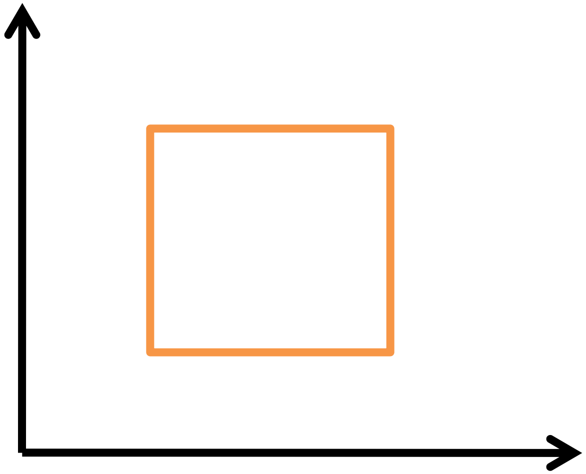
$$x' = x\cos(\alpha) - y\sin(\alpha)$$
$$y' = x\sin(\alpha) + y\cos(\alpha)$$

$$P' = R \times P$$

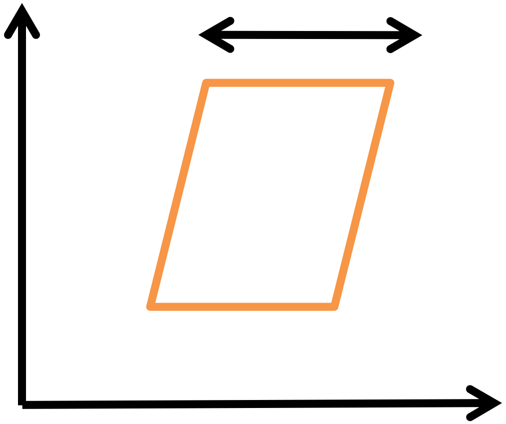
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Shearing

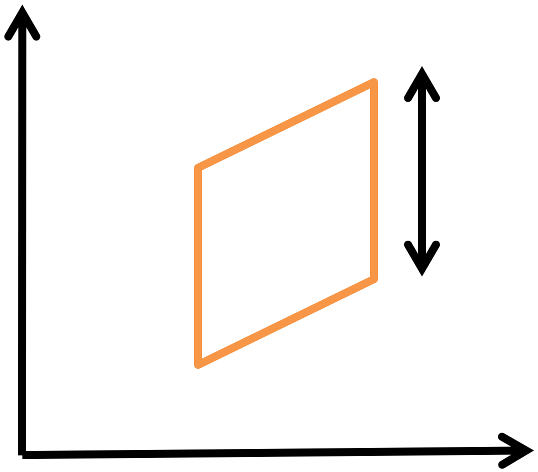
Shearing: The Idea



Original

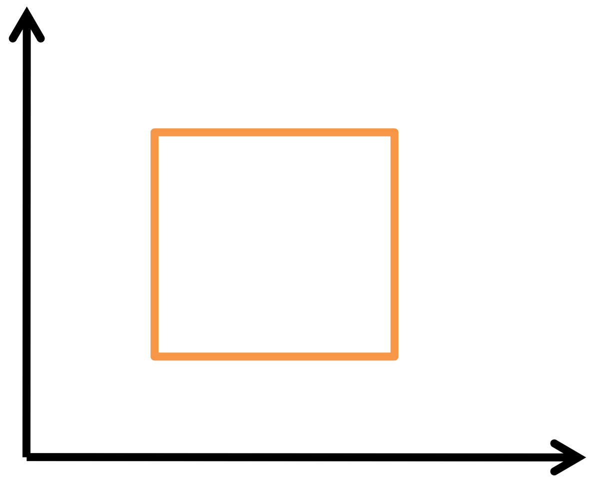


X shear

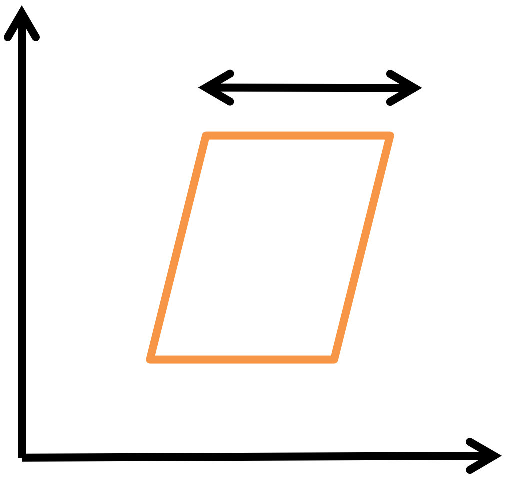


Y shear

Shearing



Original



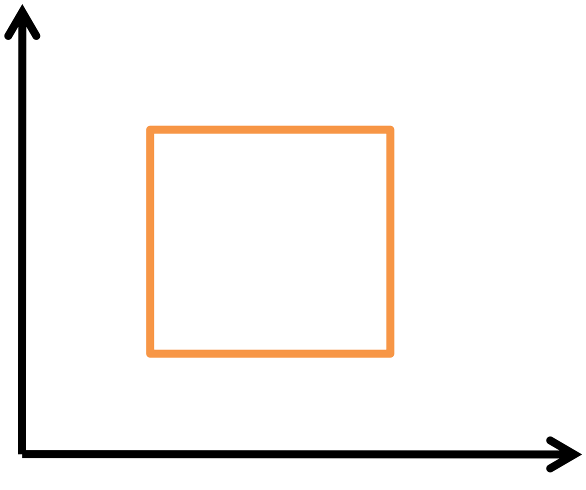
X shear

$$P' = SH_x \times P$$

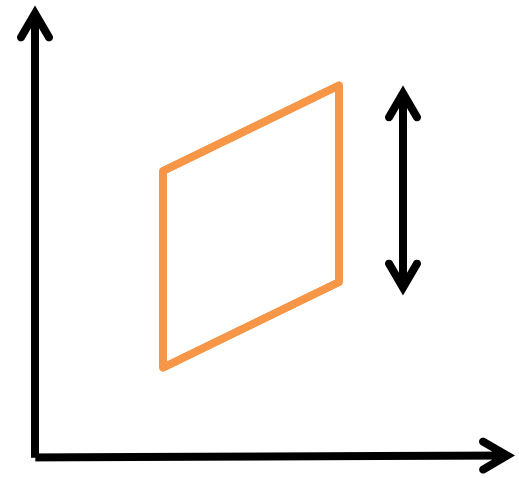
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x' = x + ay$$
$$y' = y$$

Shearing



Original



Y shear

$$P' = SH_y \times P$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

THANKS!