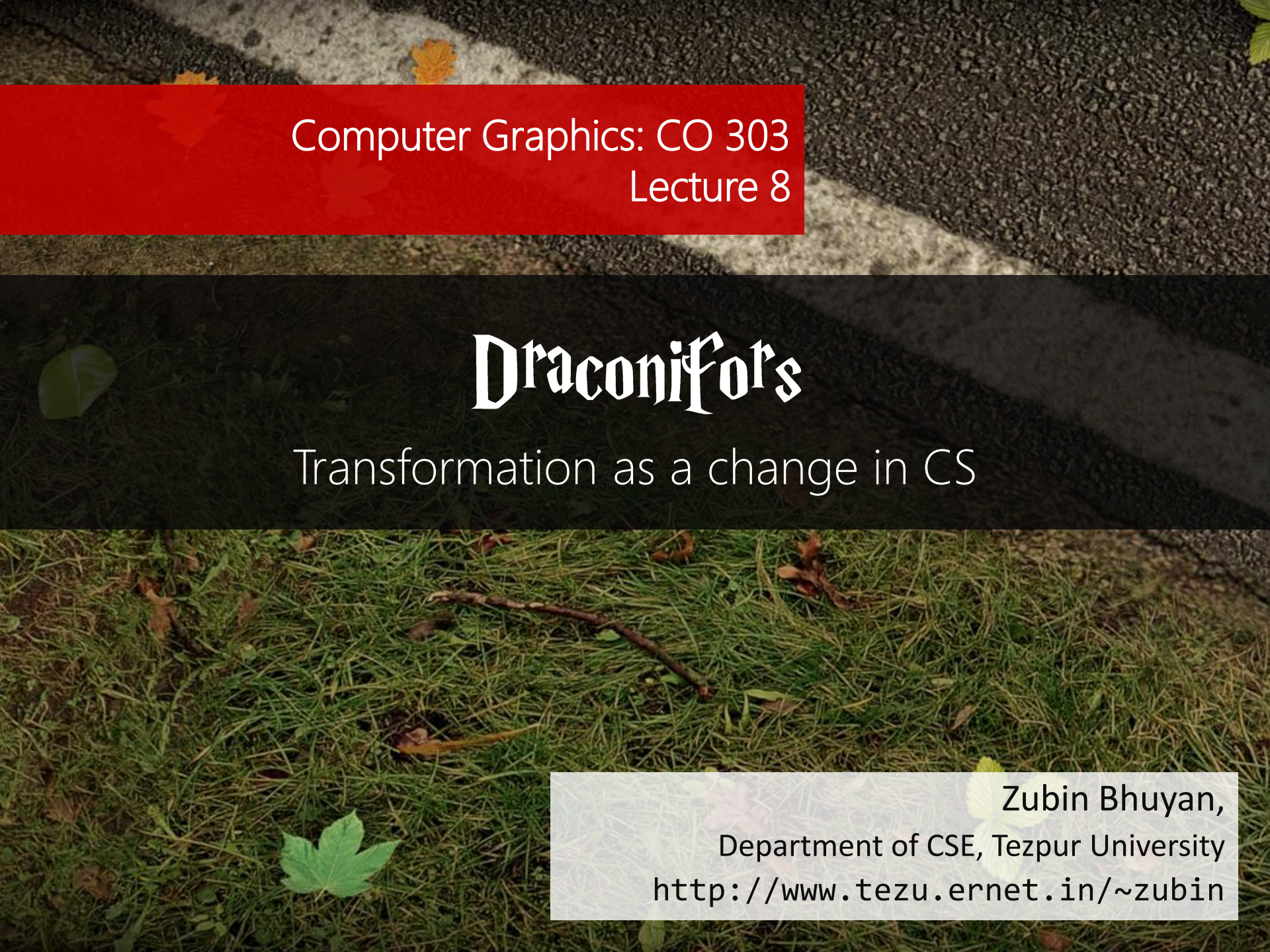

Computer Graphics: CO 303
Lecture 8

DraconiFors

Transformation as a change in CS

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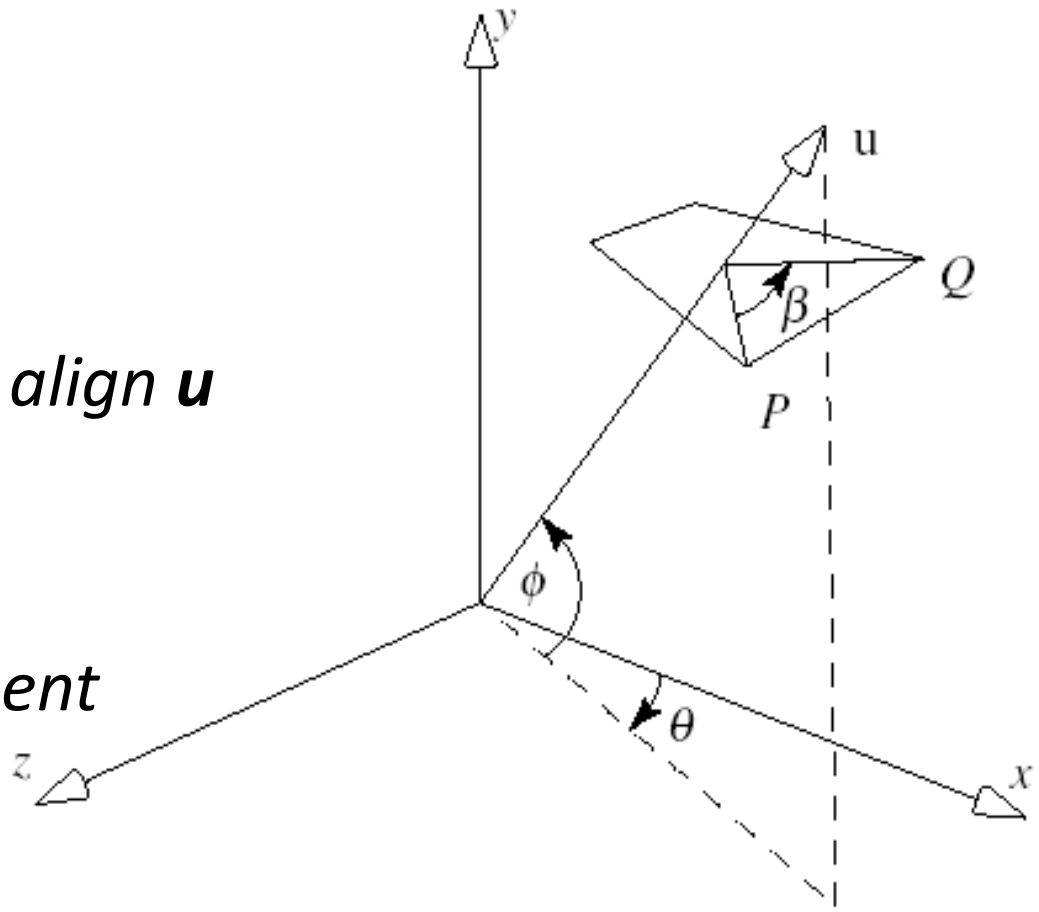
Rotation around an arbitrary axis

- Euler's theorem: Any rotation or sequence of rotations around a point is equivalent to a single rotation around an axis that passes through the point.

Rotation around an arbitrary axis

- Axis: **u**
- Point: P
- Angle: β
- Method:
  1. *Two rotations to align **u** with x-axis*
  2. *Do x-roll by β*
  3. *Undo the alignment*



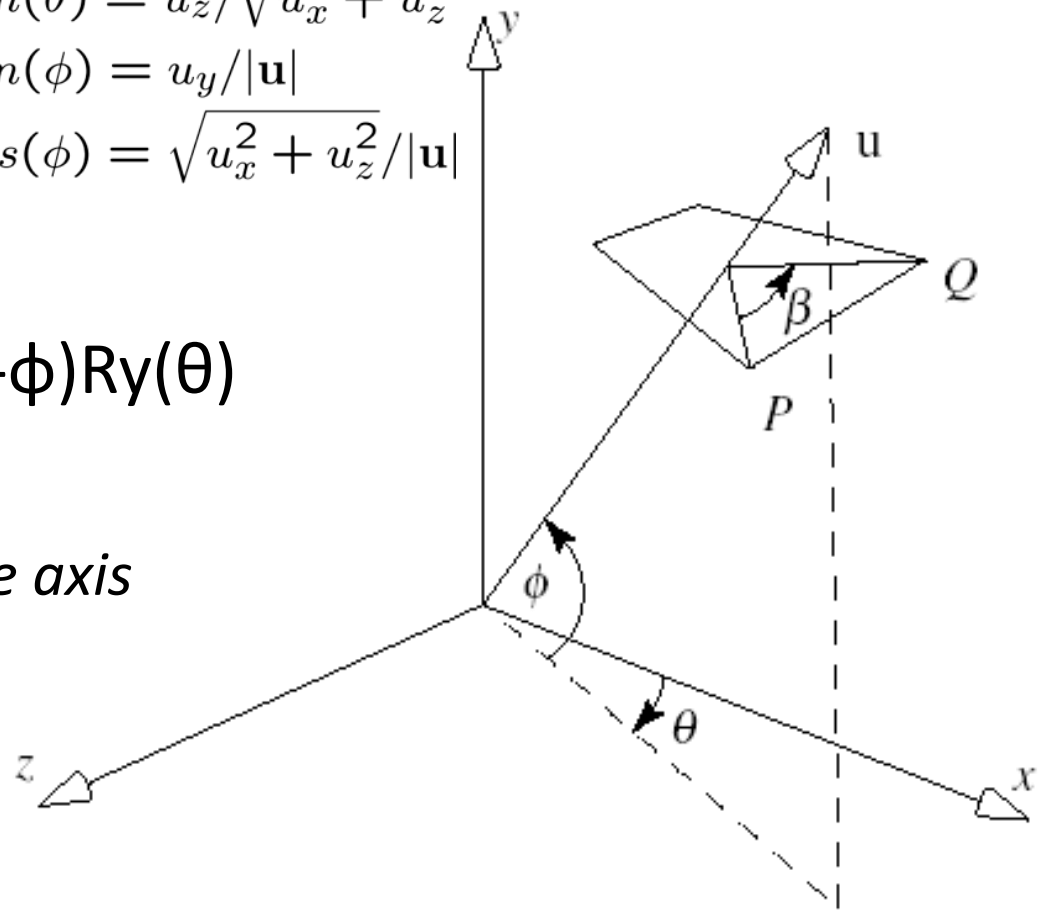
Rotation around an arbitrary axis

1. Rz(-ϕ)Ry(θ)
2. Rx(β)
3. Ry(-θ)Rz(ϕ)

$$cos(\theta) = u_x/\sqrt{u_x^2 + u_z^2}$$
$$sin(\theta) = u_z/\sqrt{u_x^2 + u_z^2}$$
$$sin(\phi) = u_y/|\mathbf{u}|$$
$$cos(\phi) = \sqrt{u_x^2 + u_z^2}/|\mathbf{u}|$$

∴ Ry(-θ)Rz(ϕ) Rx(β) Rz(-ϕ)Ry(θ)

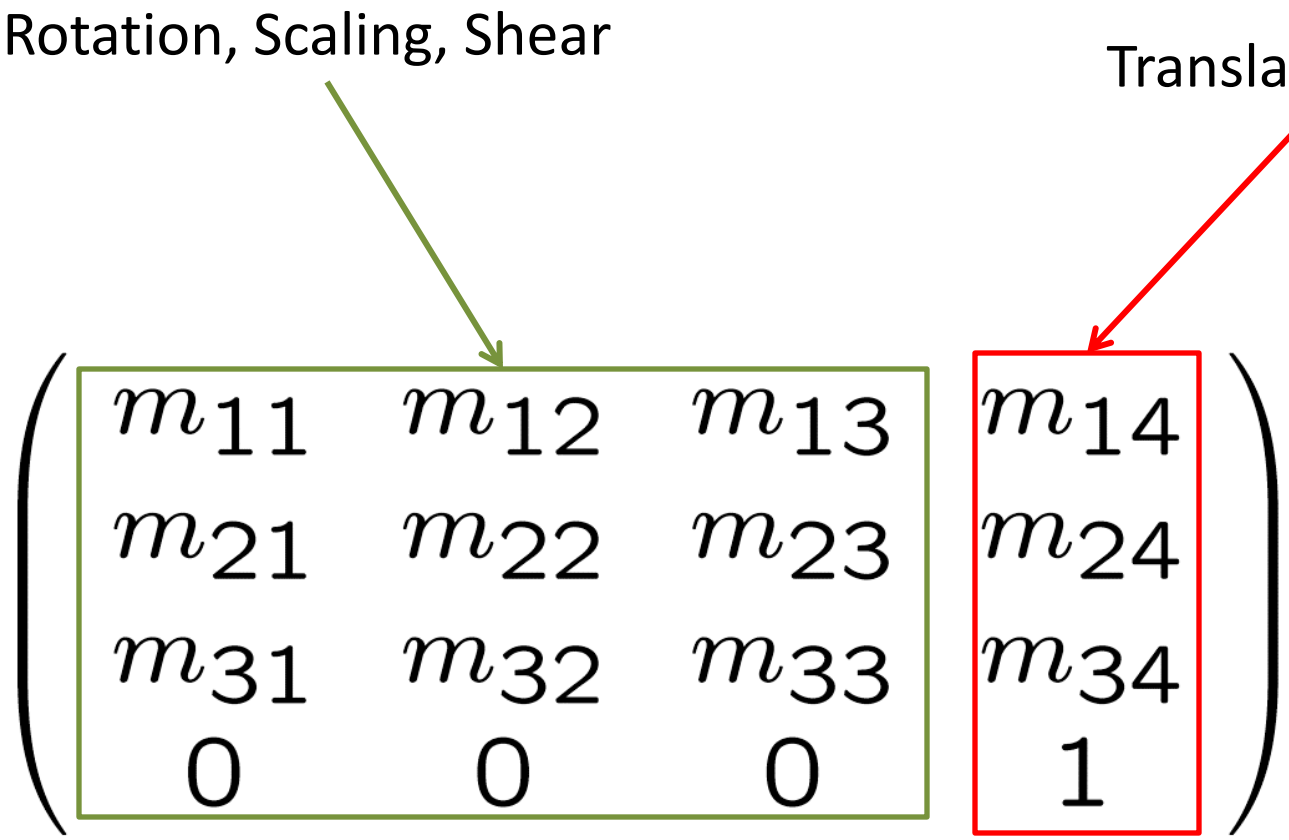
- *Add translation too if the axis is not through the origin*



Recap: Properties of affine transformations

- Preservation of affine combinations of points.
- Preservation of lines and planes.
- Preservation of parallelism of lines and planes.
- Relative ratios are preserved
- Affine transformations are composed of
- elementary ones.

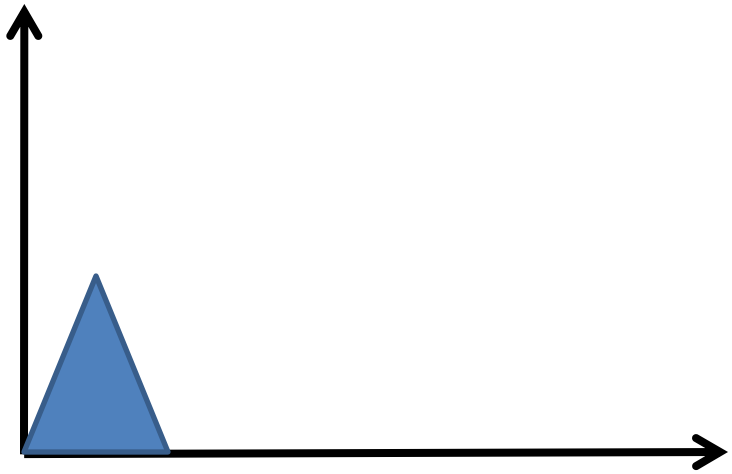
Recap: General form

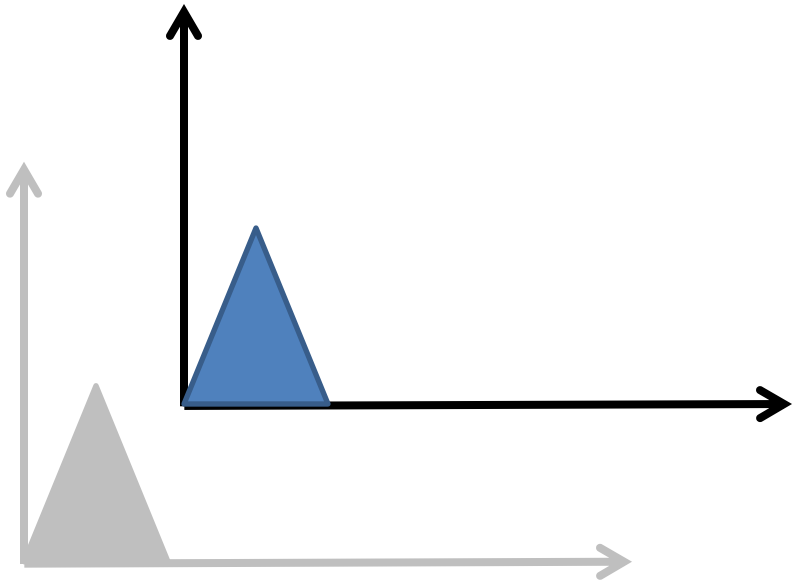


Transformations as a change in coordinate system

- All transformations we have looked at involve transforming points in a fixed coordinate system (CS).
- Can also think of them as a transformation of the CS itself.

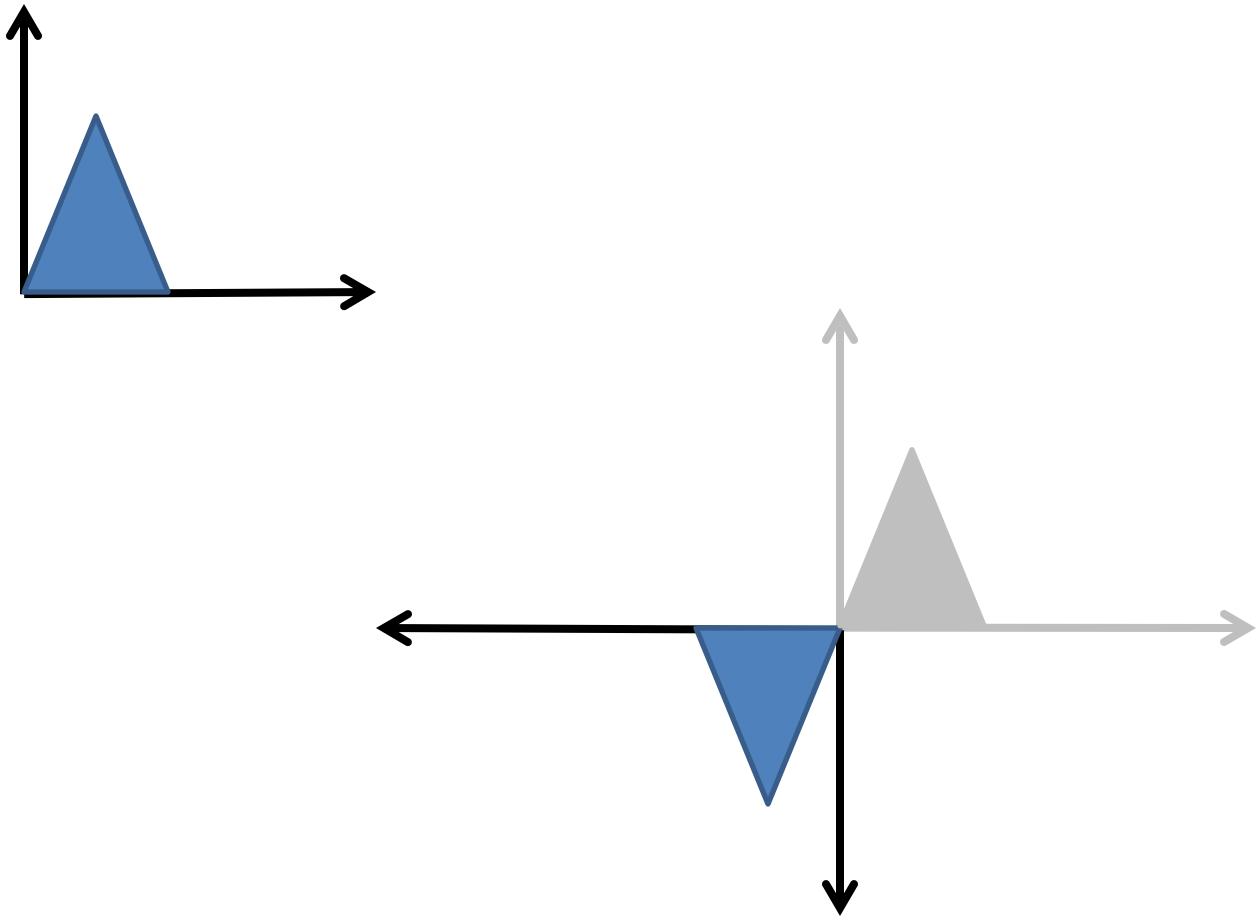
Transforming the CS





Translate(4,4)

Transforming the CS



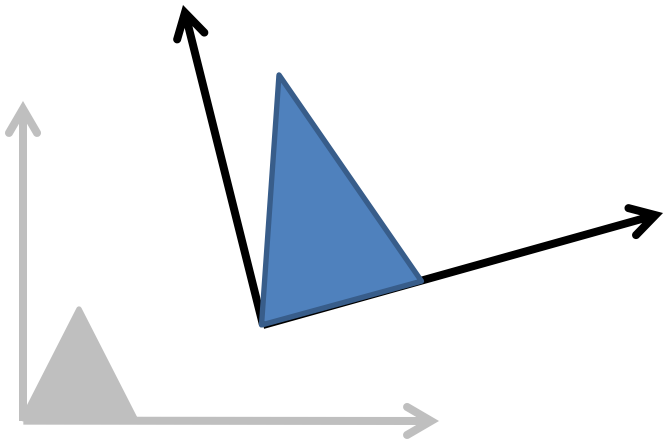
Why transform the CS?

- Objects are often defined in a “natural” or “convenient” CS.

- To draw objects transformed by T, we could:
  - Transform each vertex by T, then draw
  - Or, draw vertices in a transformed CS

Drawing in transform the CS

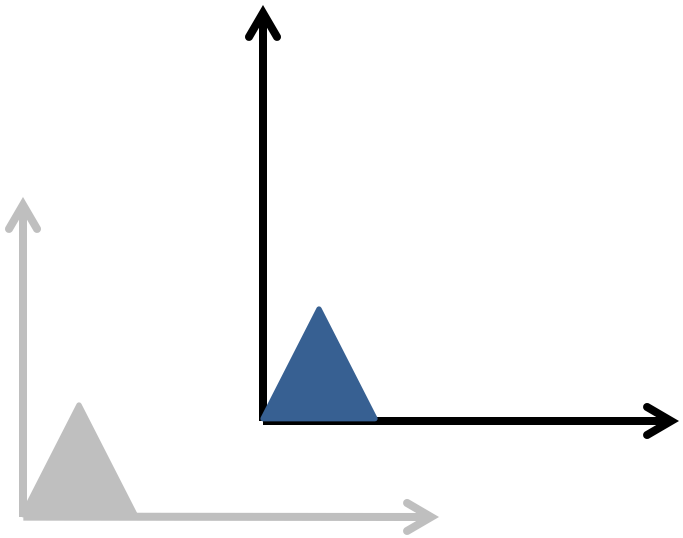
- Tell system once how to draw the object, then draw in a transformed CS to transform the object.



A triangle drawn in a CS that's been translated, rotated, and scaled.

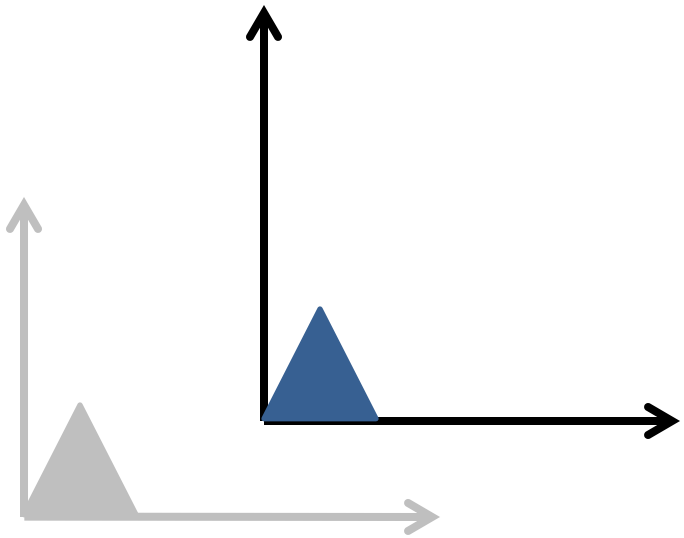
Mapping between systems

- Given:
  - The vertices of an object in CS_2
  - A transformation matrix M that transforms CS_1 to CS_2



What are the coordinates of the object's vertices in CS1?

Mapping between systems: Example



Translate(4,4)

Point P is at (0,0) in the transformed CS (CS_2). Where is it in CS_1?

Answer: (4,4)

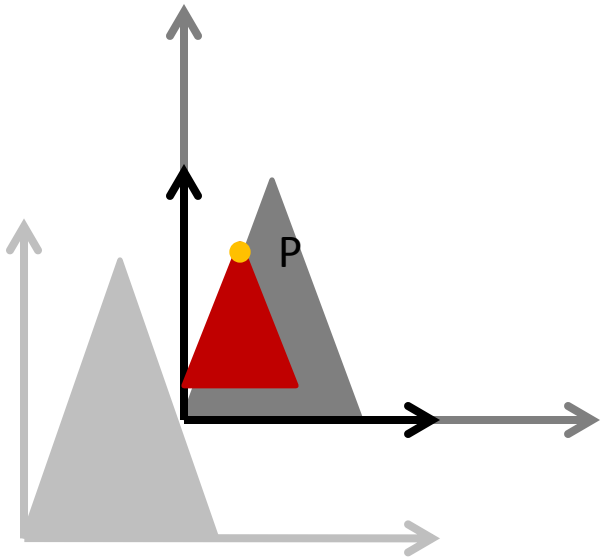
**Note: (4,4) = $T_{4,4}$ P*

Mapping Rule

- In general, if CS_1 is transformed by a matrix M to form CS_2, a point P in CS_2 is represented by MP in CS_1

Example

- In general, if CS_1 is transformed by a matrix M to form CS_2, a point P in CS_2 is represented by MP in CS_1



Translate(4,4), then
Scale(0.5, 0.5)

Where is P in CS_3? (2,2)
Where is P in CS_2? S0.5,0.5 (2,2) = (1,1)
Where is P in CS_1? T4,4 (1,1) = (5,5)

To go directly from CS3 to CS1 we can calculate:
$T_{4,4}\ S_{0.5,0.5}$ (2,2) = (5,5)

THANKS!