Computer Graphics: CO 303
Lecture 16

Spline Curves

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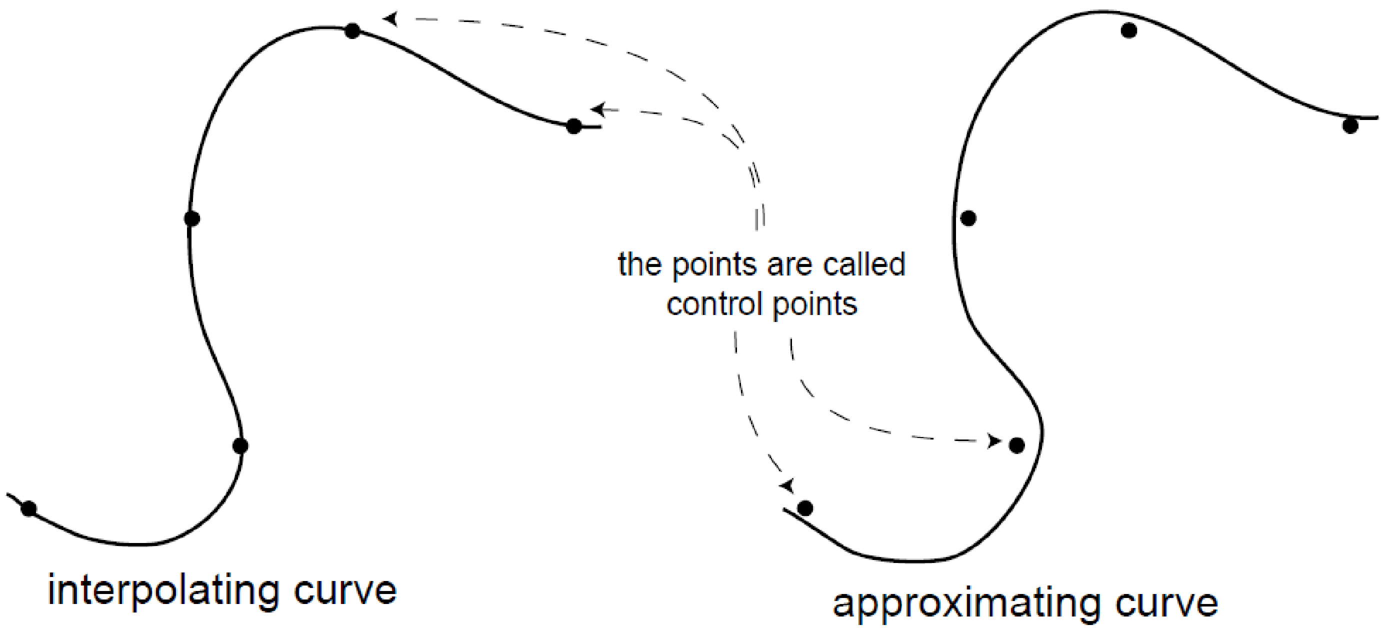
Spline curves

- A *spline curve* is a mathematical representation for which it is easy to build an interface that will allow a user to design and control the shape of complex curves and surfaces.
- General approach:
  - user enters a sequence of points (called ***control points***)
  - a curve is constructed whose shape closely follows this sequence.

Spline curves



- A curve that actually passes through each control point is called an interpolating curve.
- A curve that passes near to the control points but not necessarily through them is called an approximating curve.
- to change the shape of the curve we just move the control points.

Polynomial curves

- General form:

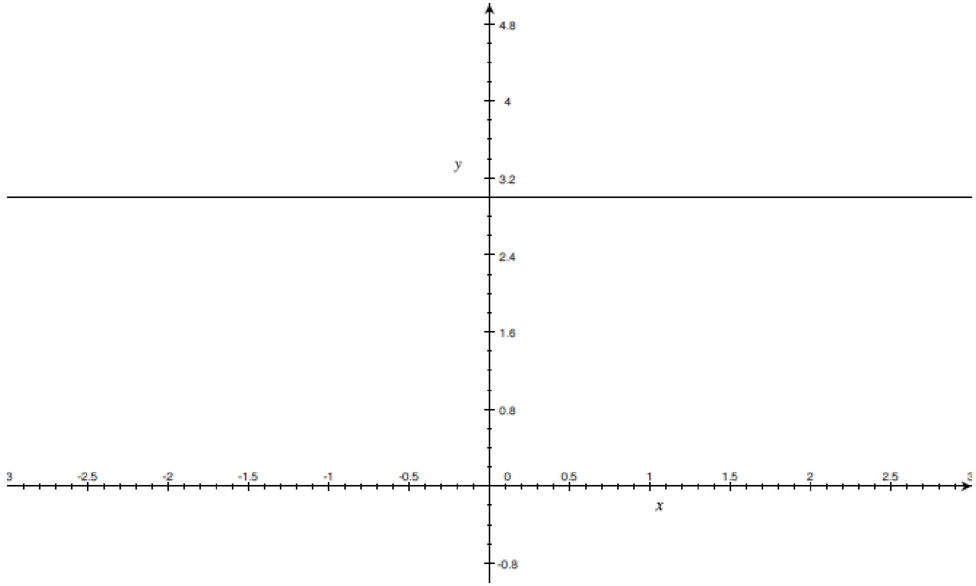
$$y = a + bx + cx^2 + dx^3 + \ldots$$

- The degree of a polynomial corresponds with the highest coecient that is non-zero.

Shapes that a polynomial make

$$y = a + bx + cx^2 + dx^3 + \ldots$$

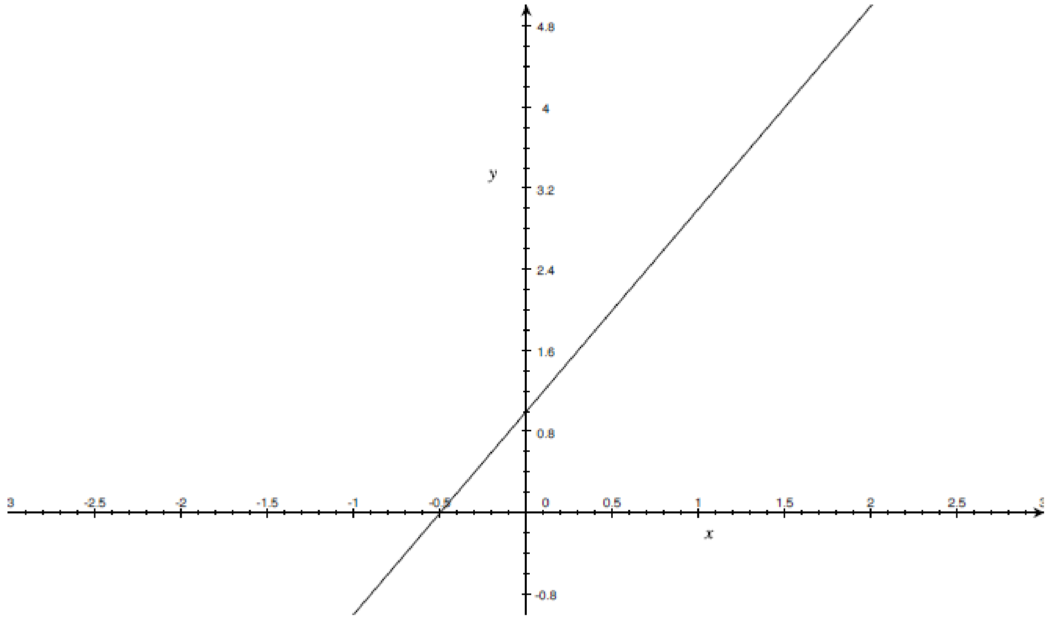
- **Degree 0**:
  - *Constant*, only a is non-zero.
  - Example: y = 3
  - A constant, uniquely defined by one point.



Shapes that a polynomial make

$$y = a + bx + cx^2 + dx^3 + \ldots$$

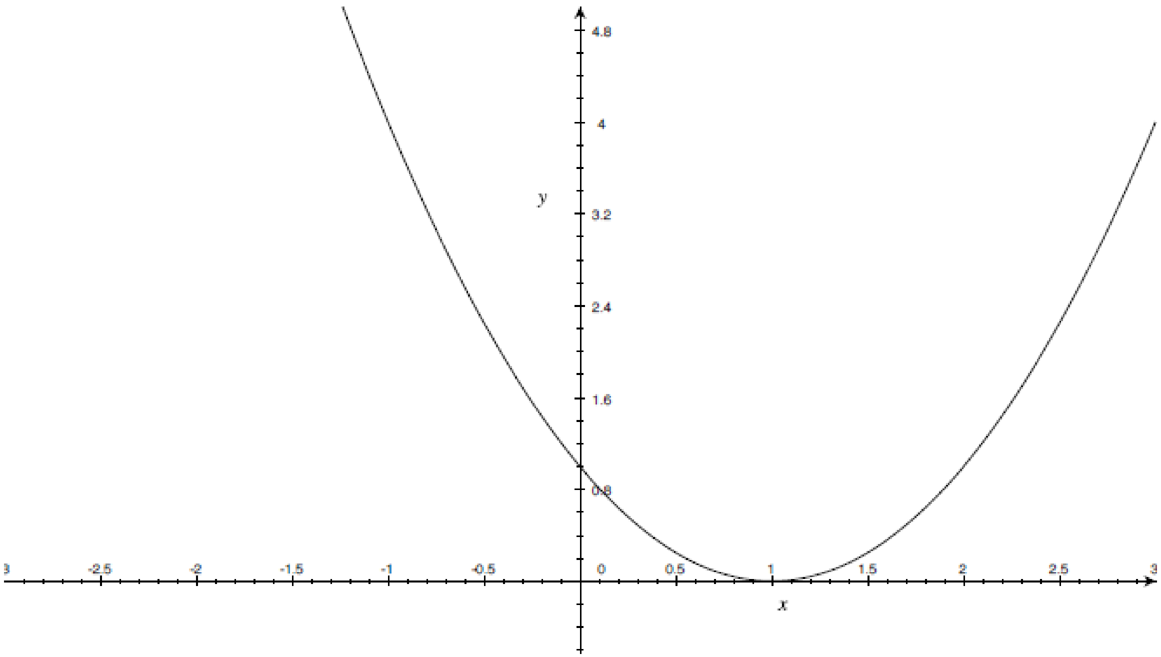
- **Degree 1**:
  - *Linear*, b is highest non-zero coecient.
  - Example: y = 1 + 2x
  - A line, uniquely dened by two points.



Shapes that a polynomial make

$$y = a + bx + cx^2 + dx^3 + \ldots$$

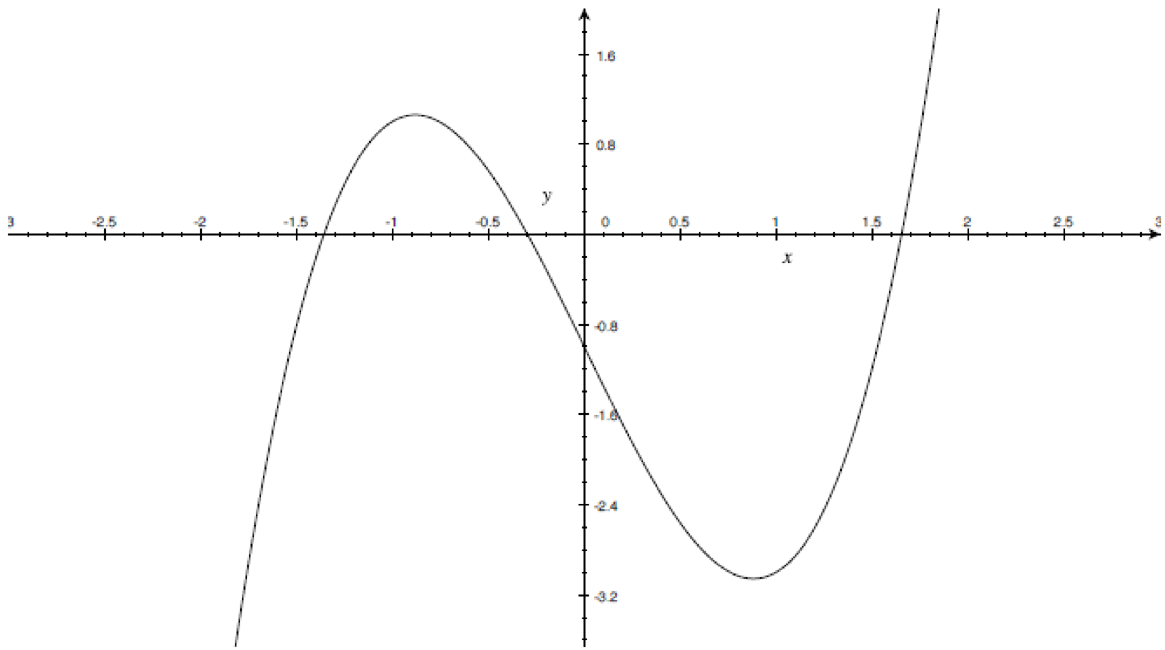
- **Degree 2**:
  - *Quadratic*, c is highest non-zero coefficient.
  - Example: $y = 1 - 2x + x^2$
  - A constant, uniquely defined by one point.



Shapes that a polynomial make

$$y = a + bx + cx^2 + dx^3 + \ldots$$

- **Degree 3**:
  - *Cubic*, d is highest non-zero coecient.
  - Example: y = -1 – 7/2x + 3/ 2x^3
  - A constant, uniquely defined by one point.



Cubic Polynomial

- Most typically chosen for constructing smooth curves in computer graphics.
- It is used because:
  - it is the lowest degree polynomial that can support an inflection.
  - it is very well behaved numerically, ie, the curves will usually be smooth.

Cubic Polynomial

- The user enters four control points,
- and the program solves for the four coefficients a; b; c and d which cause the polynomial to pass through the four control points.

Cubic Polynomial

general form: $a + bx + cx^2 + dx^3 = y$

point $(-1, 2)$**:** $a - b + c - d = 2$

point $(0, 0)$**:** $a = 0$

point $(1, -2)$**:** $a + b + c + d = -2$

point $(2, 0)$**:** $a + 2b + 4c = 8d = 0$

This can be written in matrix form

$$\begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -2 \\ 0 \end{bmatrix}$$

The solution is

$$\mathbf{a} = M^{-1}\mathbf{y},$$

Point of inflection

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Epigraph

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THANKS!