Computer Graphics: CO 303

Lecture 2

Engorgio

Projections

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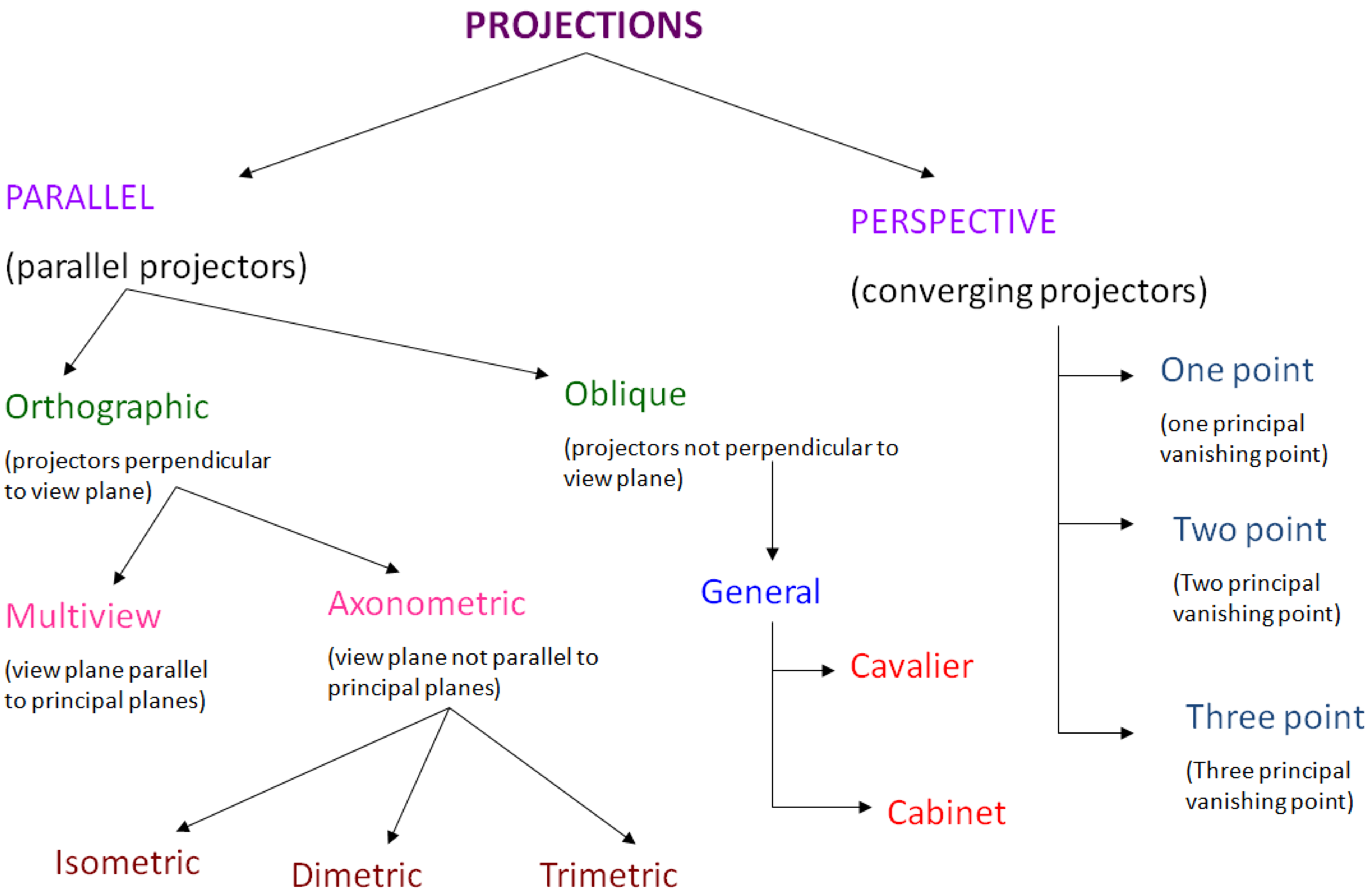
Road in perspective

Taxonomy

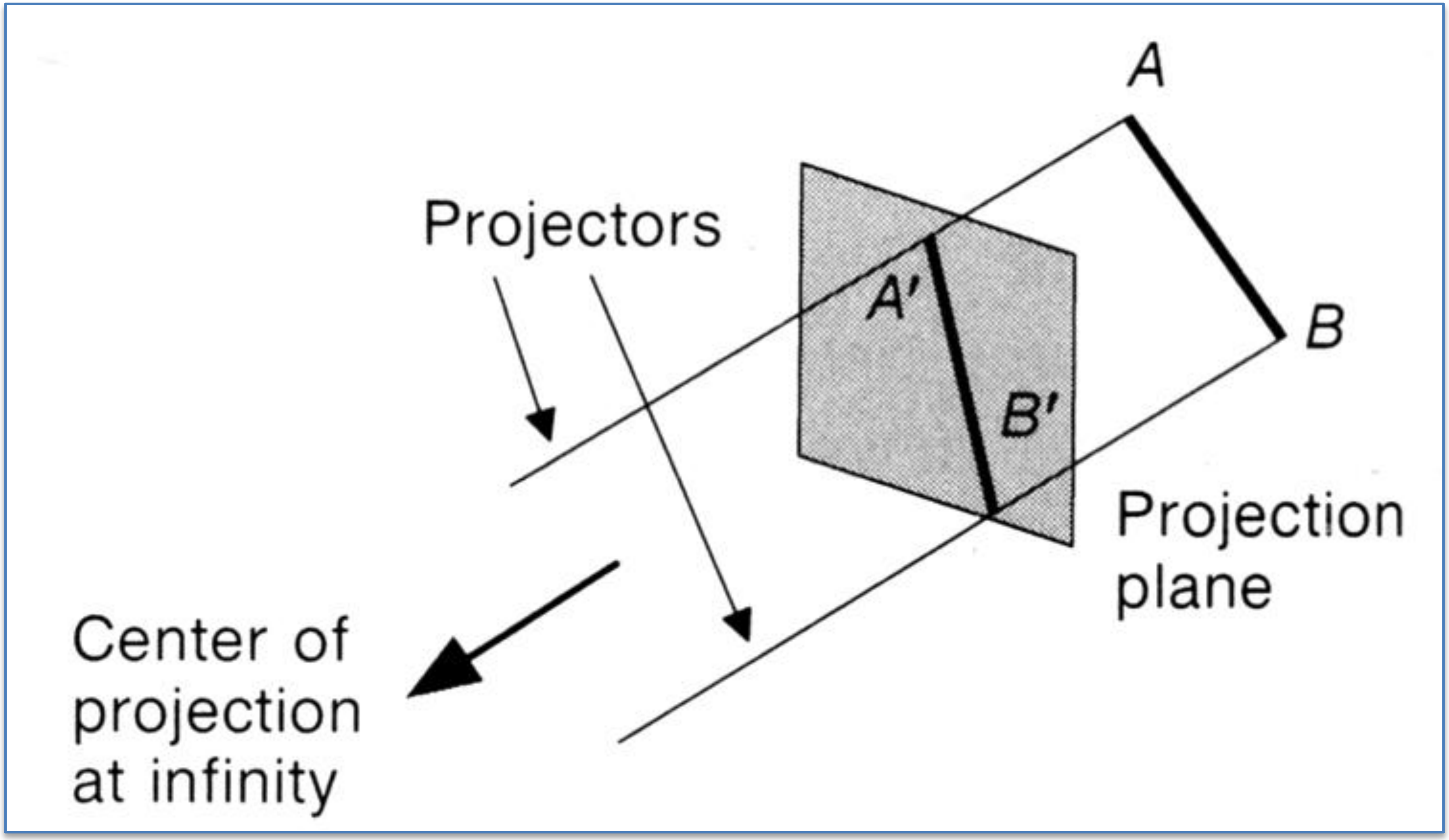


PROJECTIONS
PARALLEL
(parallel projectors)
PERSPECTIVE
(converging projectors)
Orthographic
(projectors perpendicular to view plane)
Oblique
(projectors not perpendicular to view plane)
One point
(one principal vanishing point)
Two point
(Two principal vanishing point)
Three point
(Three principal vanishing point)
Multiview
(view plane parallel to principal planes)
Axonometric
(view plane not parallel to principal planes)
General
Cavalier
Cabinet
Isometric
Dimetric
Trimetric

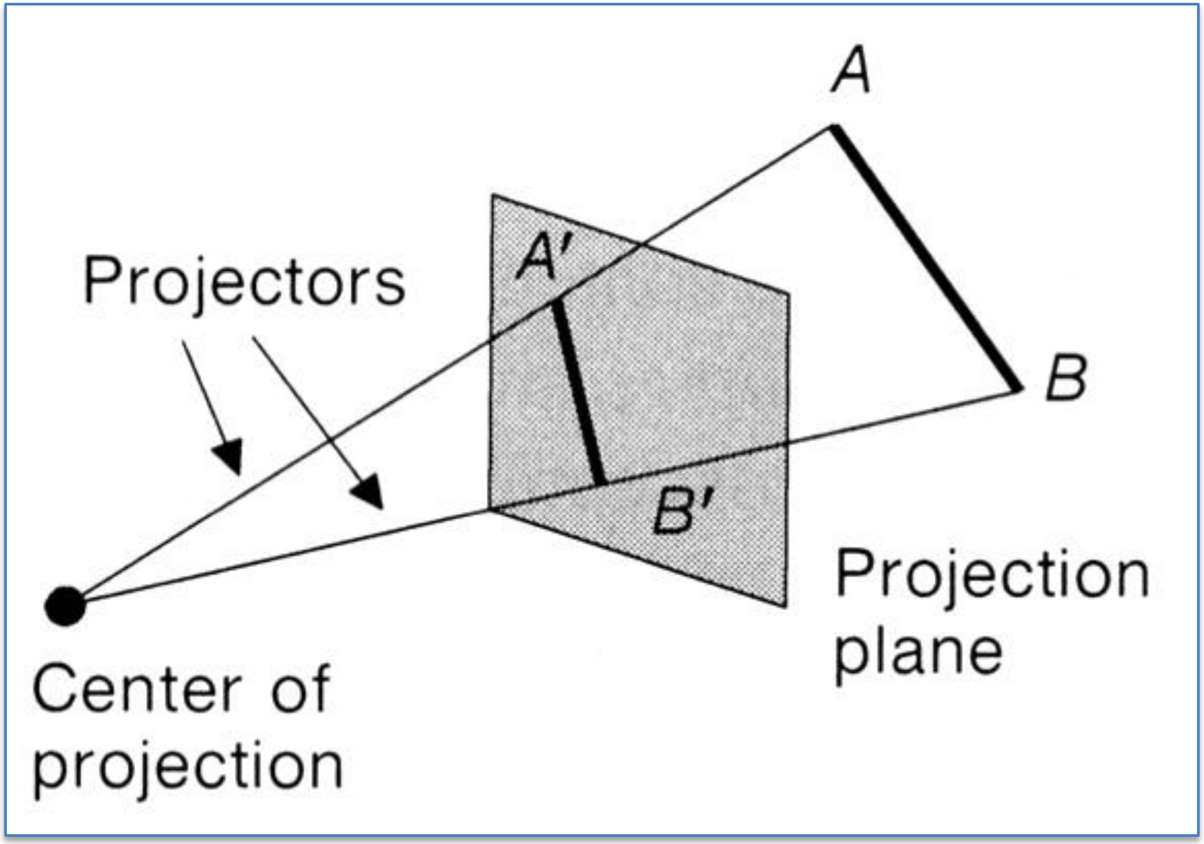
2 types of projections

- **2 types of projections**
  - *Perspective :* In **perspective projection**, object position are transformed to the view plane along lines that converge to a point called **projection reference point (center of projection)**
  - *Parallel:* In **parallel projection**, coordinate positions are transformed to the view plane along parallel lines.

Parallel Projection

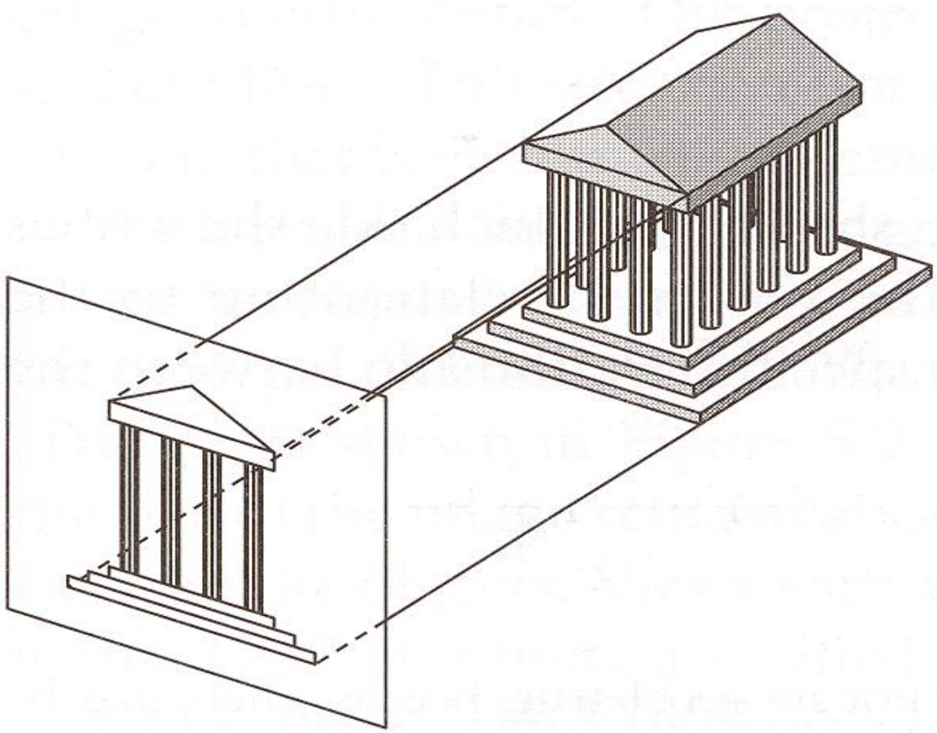


Perspective Projection



Parallel Projection

- Center of projection is at infinity
  - Direction of projection (DOP) same for all points



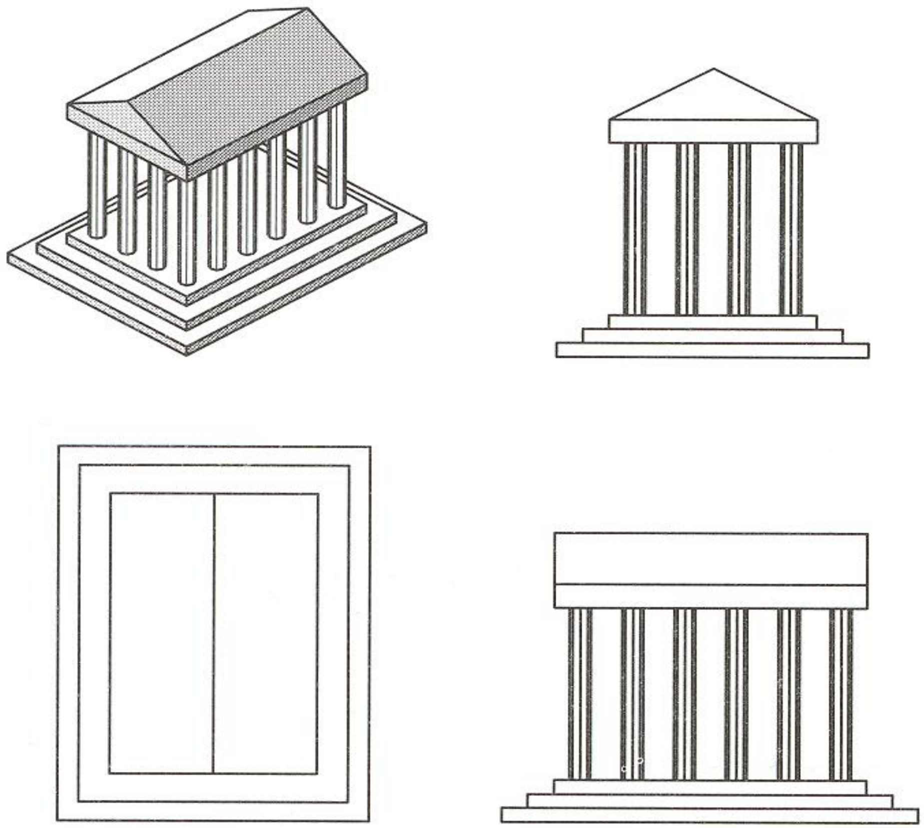
Parallel Projection

2 types:

- **Orthographic** : when the projection is perpendicular to the view plane. In short,
  - direction of projection = normal to the projection plane.
  - the projection is perpendicular to the view plane.
- **Oblique** : when the projection is not perpendicular to the view plane. In short,
  - direction of projection ≠ normal to the projection plane.
  - Not perpendicular.

Orthographic (or orthogonal) Projections

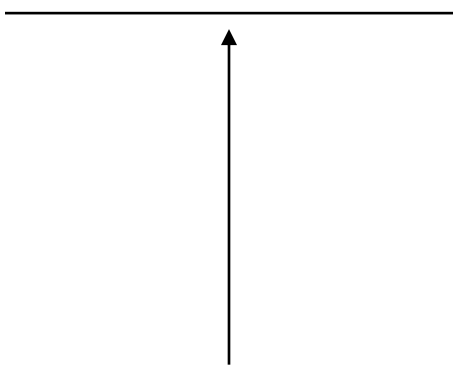
- DOP perpendicular to view plane



Top Side

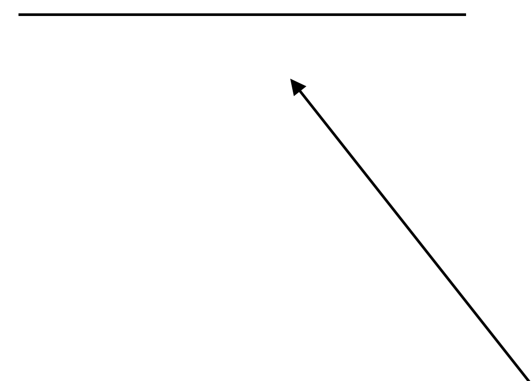
Orthographic Projections

- Orthographic projection



when the projection is perpendicular to the view plane

Oblique projection

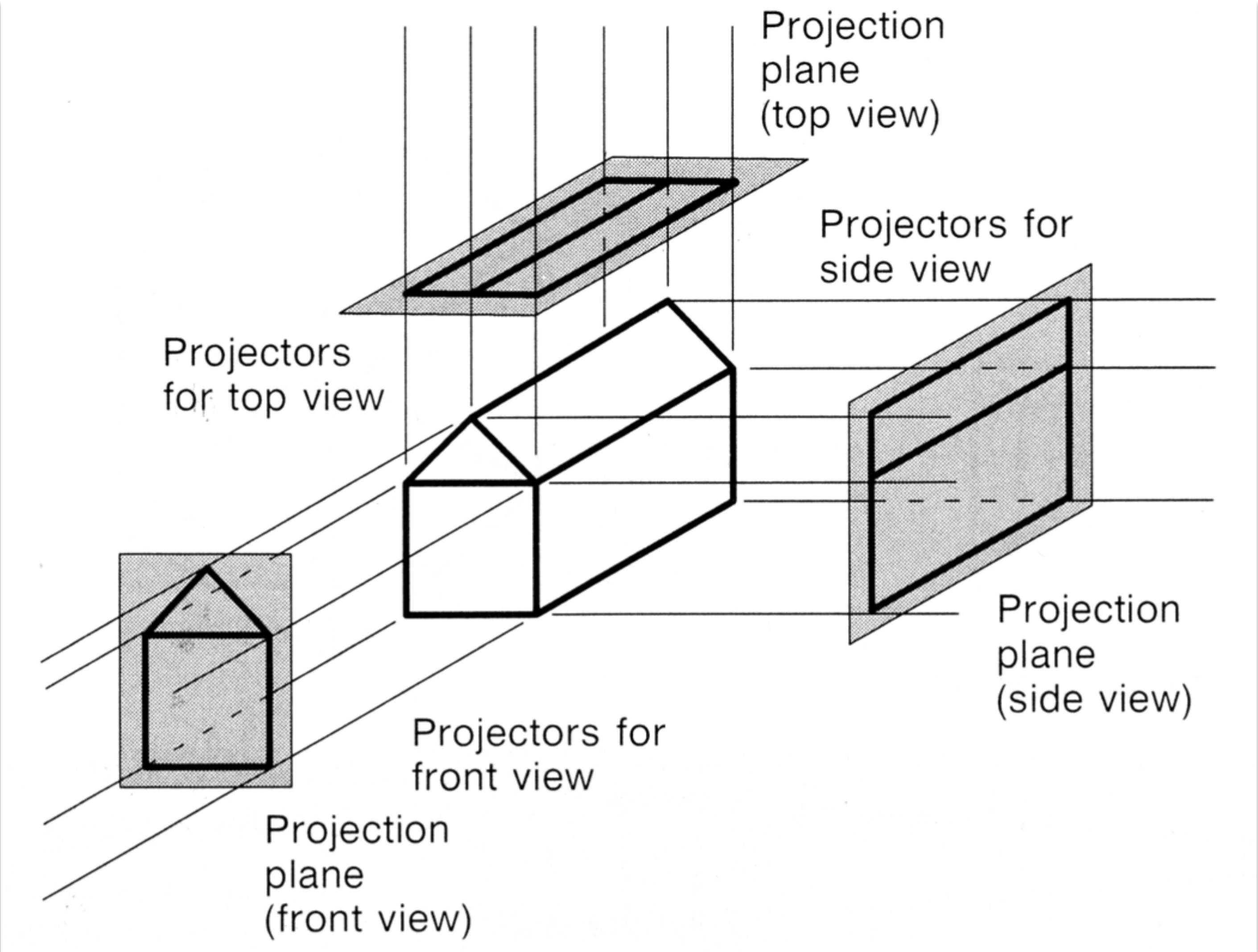


when the projection is not perpendicular to the view plane

Orthographic (or orthogonal) Projections

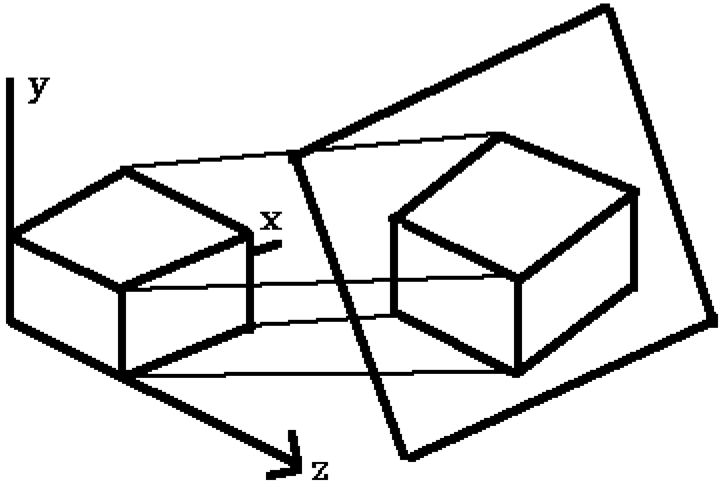
- Front, side and rear orthographic projection of an object are called **elevations** and the top orthographic projection is called **plan view**.
- all have projection plane perpendicular to a principle axes.
- Here length and angles are accurately depicted and measured from the drawing, so engineering and architectural drawings commonly employee this.
  - However, As only one face of an object is shown, it can be hard to create a mental image of the object, even when several views are available.

Orthographic (or orthogonal) Projections



Orthographic (or orthogonal) Projections

- Orthographic projections that *show more than one face of an object* are called **axonometric** orthographic projections.
- The most common axonometric projection is an **isometric** projection where the projection plane intersects each coordinate axis in the model coordinate system at an equal distance.



Orthographic (or orthogonal) Projections

- Simple Orthographic Transformation

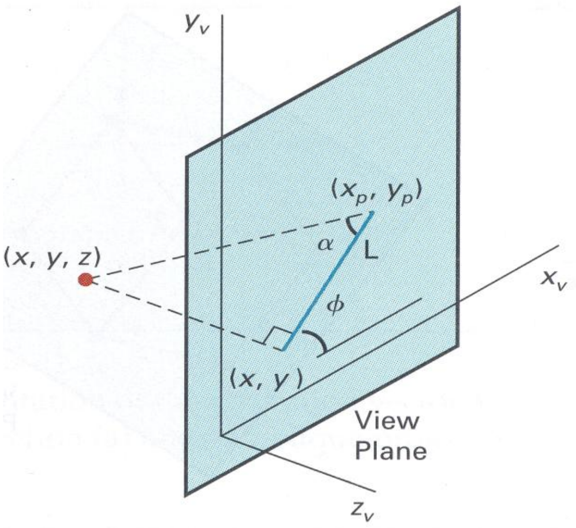
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

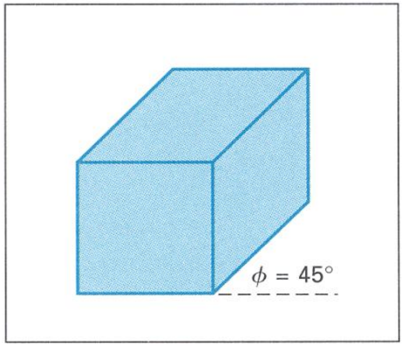
- Original world units are preserved

Oblique parallel projections

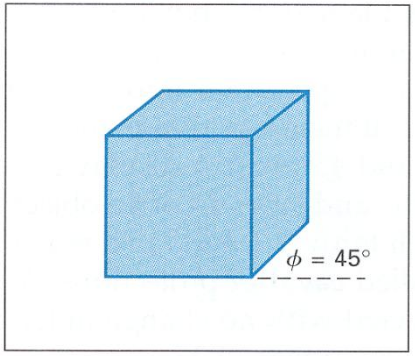
- DOP not perpendicular to view plane

- 2 common oblique parallel projections:
  - *Cavalier and Cabinet*





Cavalier
(DOP $\alpha = 45^o$)
$\tan(\alpha) = 1$



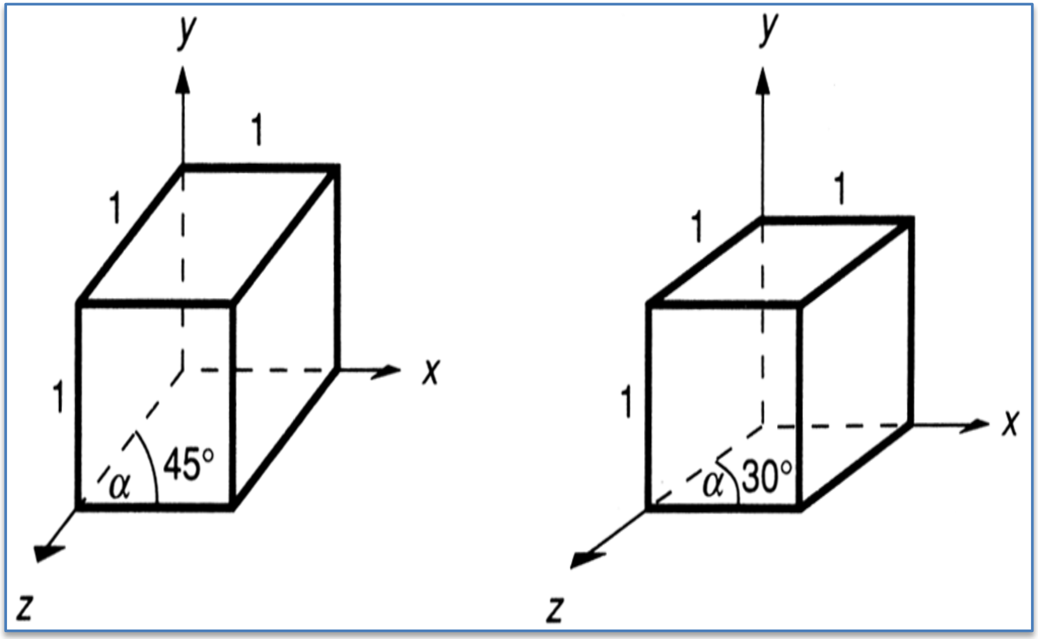
Cabinet
(DOP $\alpha = 63.4^o$)
$\tan(\alpha) = 2$

Oblique parallel projections

- 2 common oblique parallel projections:
  - *Cavalier and Cabinet*

Cavalier projection:

All lines perpendicular to the projection plane are projected with no change in length.



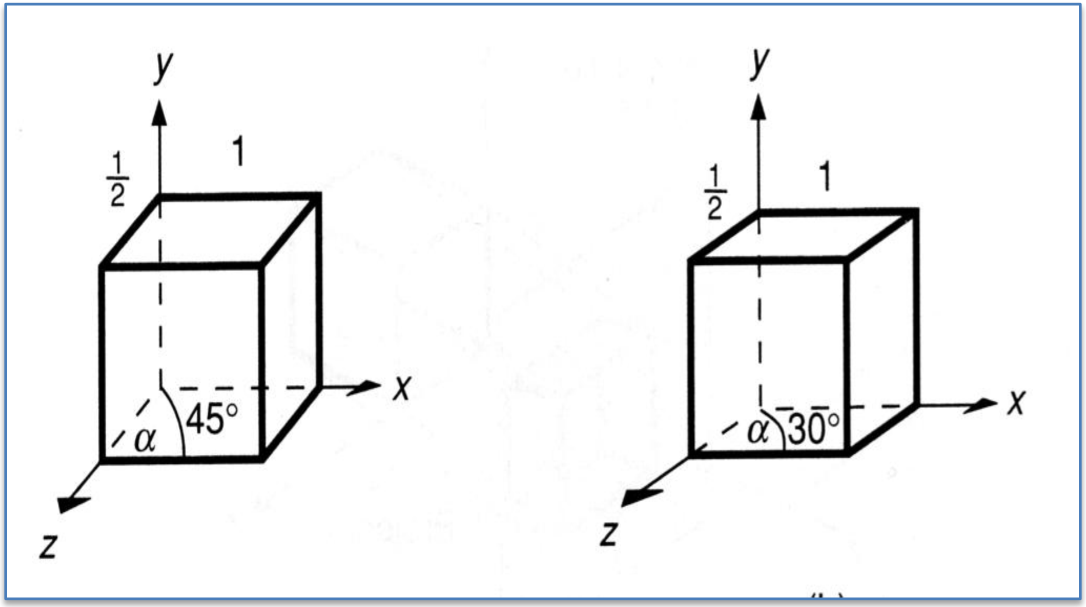
Oblique parallel projections

- 2 common oblique parallel projections:
  - *Cavalier and Cabinet*

Cabinet projection:

– Lines which are perpendicular to the projection plane (viewing surface) are projected at 1 / 2 the length .

– This results in foreshortening of the z axis, and provides a more "realistic" view.



Perspective Projection

Perspective Projection

- Characterstics:
  - Center of Projection (CP) is a finite distance from object
  - Projectors are rays (i.e., non-parallel)
  - Vanishing points
  - Objects appear smaller as distance from CP (eye of observer) increases
  - Difficult to determine exact size and shape of object
  - Most realistic, difficult to execute

Perspective Projection

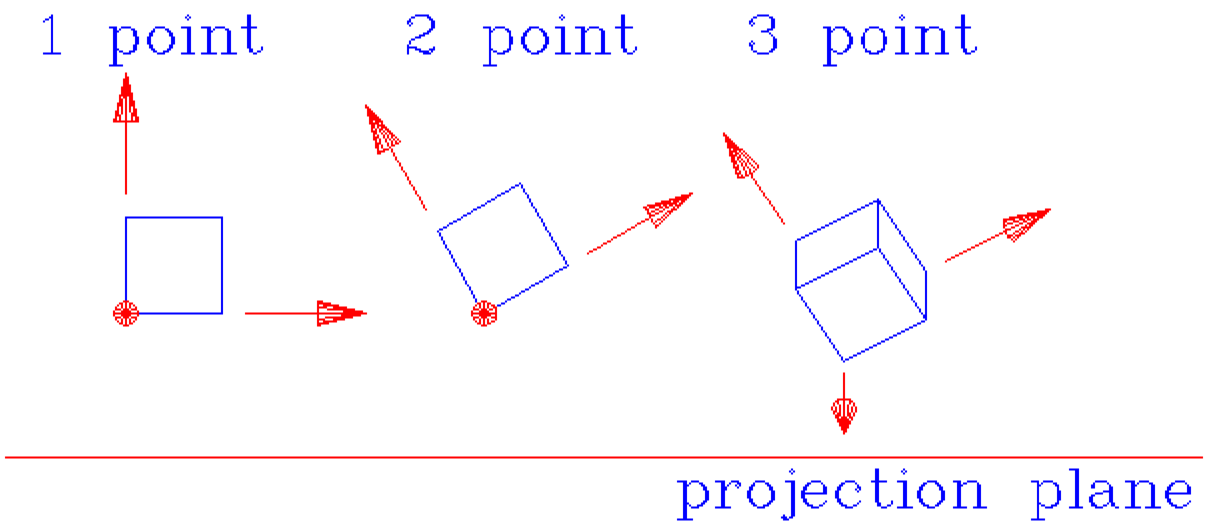
- When a 3D object is projected onto view plane using perspective transformation equations, any set of parallel lines in the object that are *not* parallel to the projection plane, converge at a vanishing point.
  - There are an infinite number of vanishing points, depending on how many set of parallel lines there are in the scene.
- If a set of lines are parallel to one of the three principle axes, the vanishing point is called an *principal vanishing point*.
  - There are at most 3 such points, corresponding to the number of axes cut by the projection plane.

Vanishing Points

- Certain set of parallel lines appear to meet at a different point
  - The *Vanishing point* for this direction
- Principal vanishing points are formed by the apparent intersection of lines parallel to one of the three principal x, y, z axes.
- The number of principal vanishing points is determined by the number of principal axes intersected by the view plane.
- Sets of parallel lines on the same plane lead to *collinear* vanishing points.
  - The line is called the *horizon* for that plane

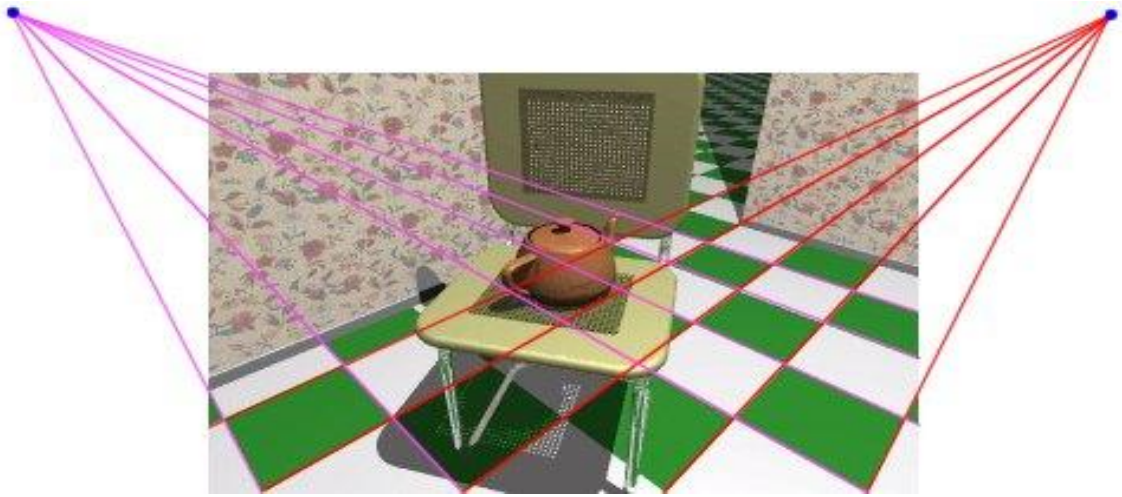
Classes of Perspective Projection

- One-Point Perspective
- Two-Point Perspective
- Three-Point Perspective



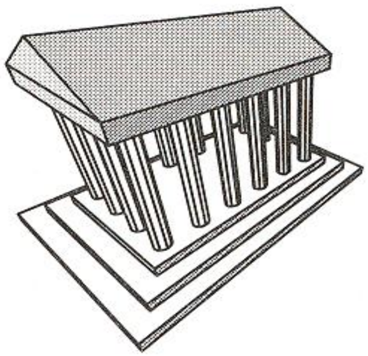
Perspective Transformation

- First discovered by Donatello, Brunelleschi, and DaVinci during Renaissance
- Objects closer to viewer look larger
- Parallel lines appear to converge to single point

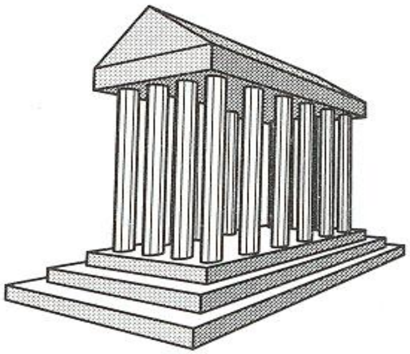


Perspective Transformation

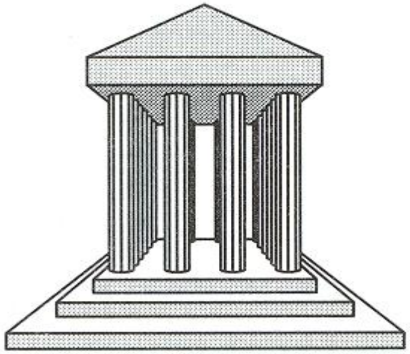
- How many vanishing points?



3-Point
Perspective

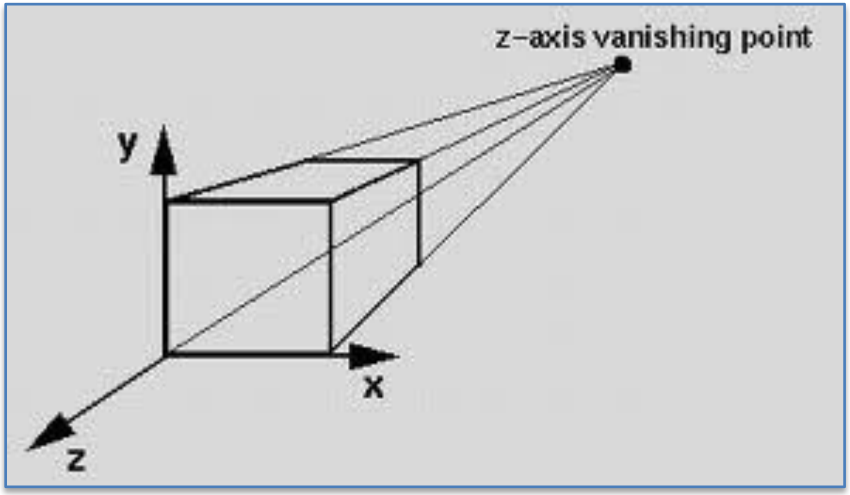


2-Point
Perspective



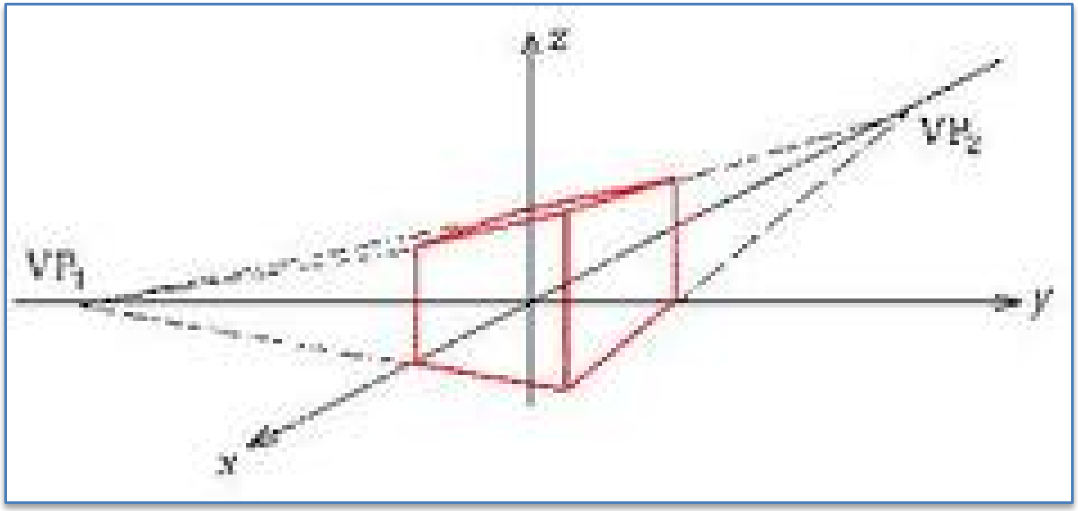
1-Point
Perspective

One-Point Perspective



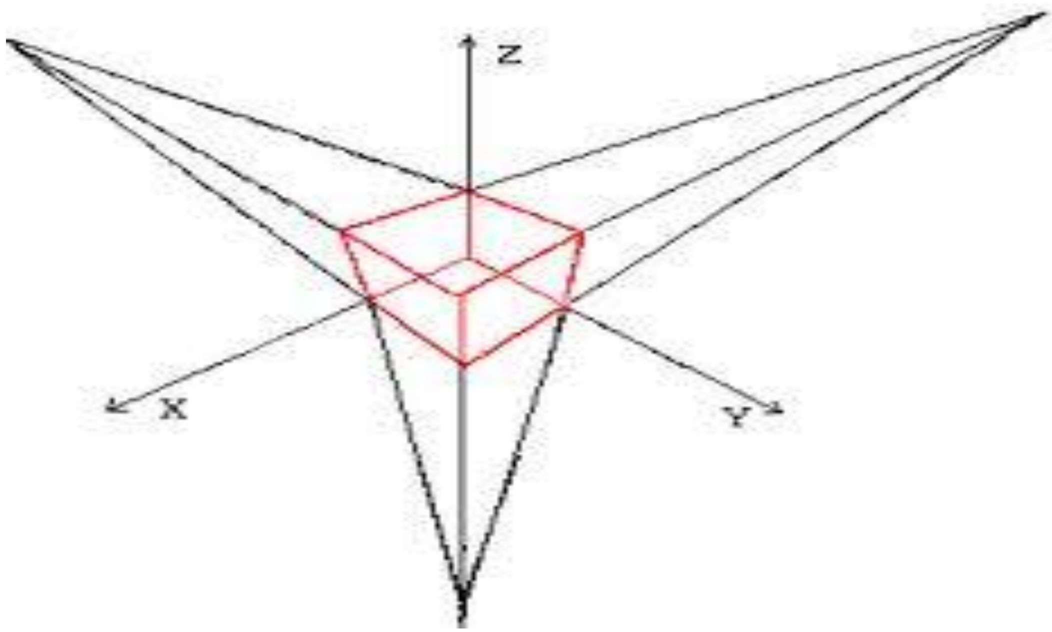
Two-point perspective projection

This is often used in architectural, engineering and industrial design drawings.



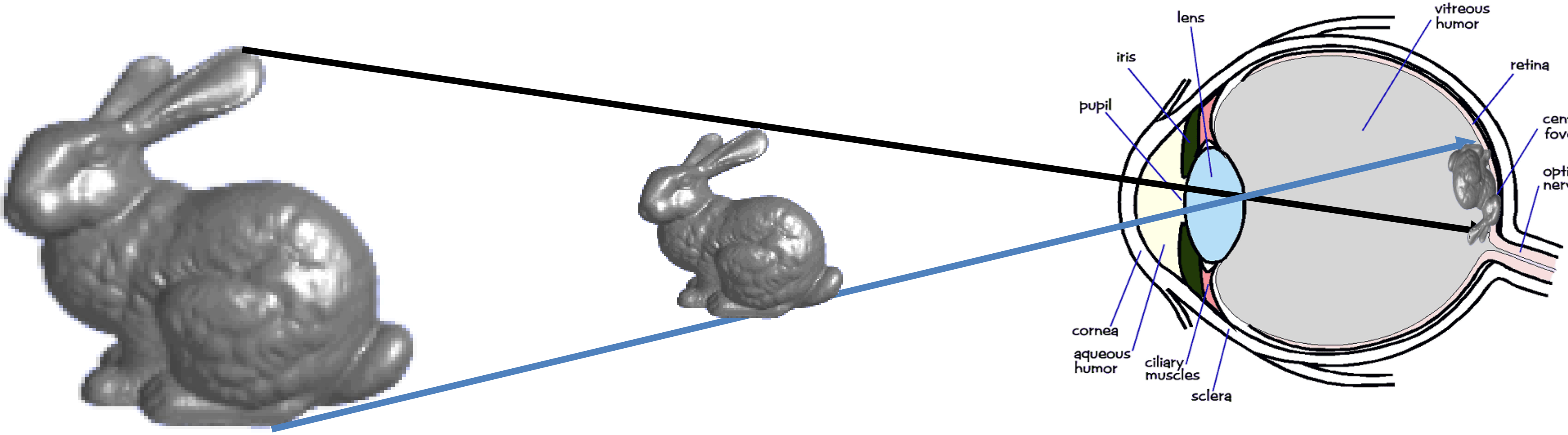
Three-point perspective projection

Three-point perspective projection is used less frequently as it adds little extra realism to that offered by two-point perspective projection



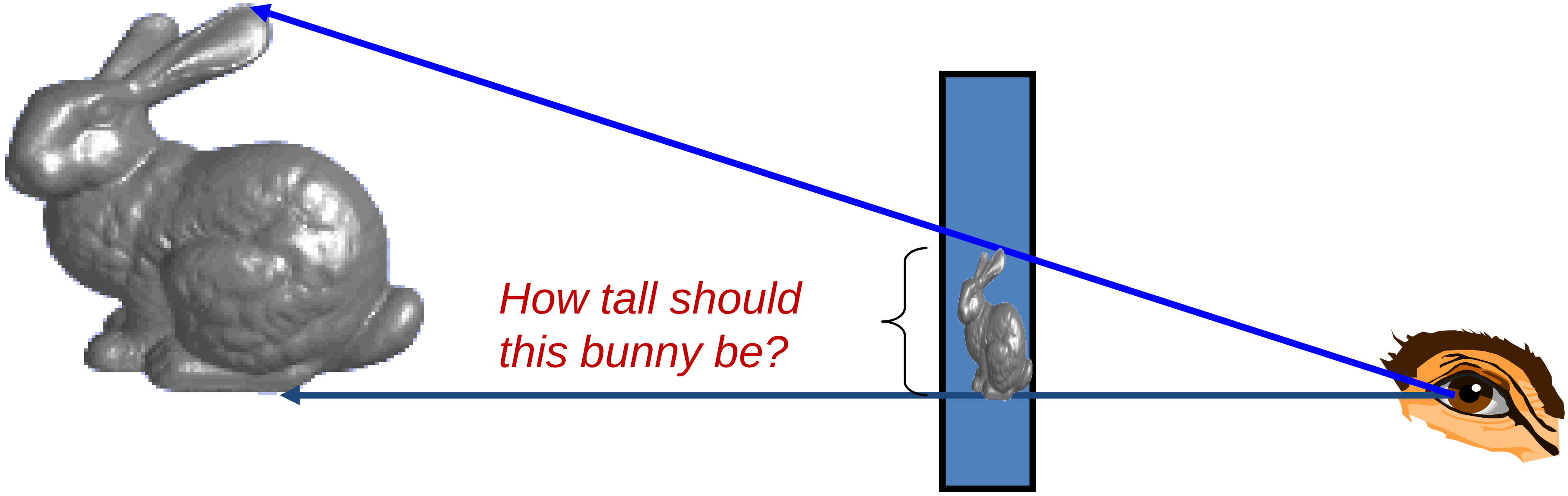
Perspective projection

- In the real world, objects exhibit perspective foreshor: distant objects appear smaller
- The basic situation:



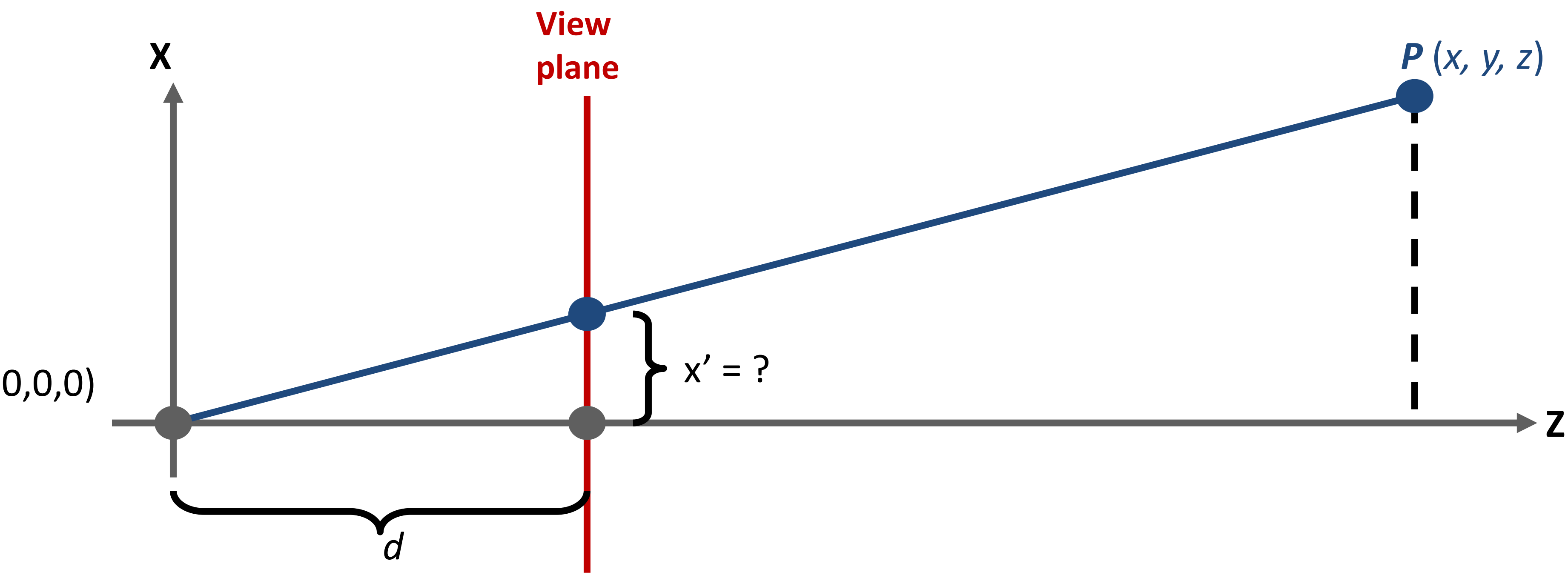
Perspective projection

When we do 3-D graphics, we think of the screen as a 2-D window onto the 3-D world:



Perspective Projection

- The geometry of the situation is that of similar triangles. View from above:



- What is x' ?

Perspective Projection

- Desired result for a point $[x, y, z, 1]^T$ projected onto the view plane:

$$\frac{x'}{d} = \frac{x}{z}, \quad \frac{y'}{d} = \frac{y}{z}$$

$$x' = \frac{d \cdot x}{z} = \frac{x}{z/d}, \quad y' = \frac{d \cdot y}{z} = \frac{y}{z/d}, \quad z = d$$

- What could a matrix look like to do this?

Perspective Projection Matrix

- Answer:

$$M_{perspective} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$

Perspective Projection Matrix

- Example:

$$\begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

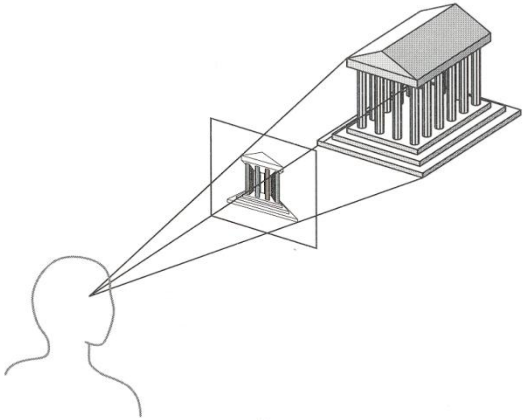
Or, in 3-D coordinates: $\left(\frac{x}{z/d}, \quad \frac{y}{z/d}, \quad d \right)$

Perspective Projection Matrix

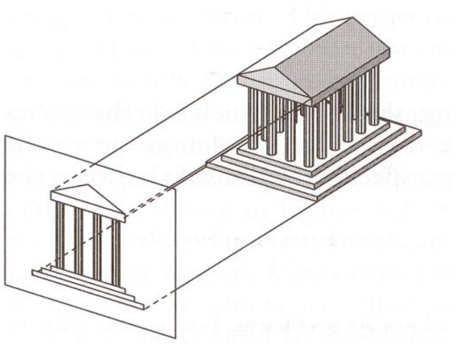
- Now that we can express perspective foreshortening as a matrix, we can compose it onto our other matrices with the usual matrix multiplication
- End result: a single matrix encapsulating modeling, viewing, and projection transforms

Perspective vs. Parallel

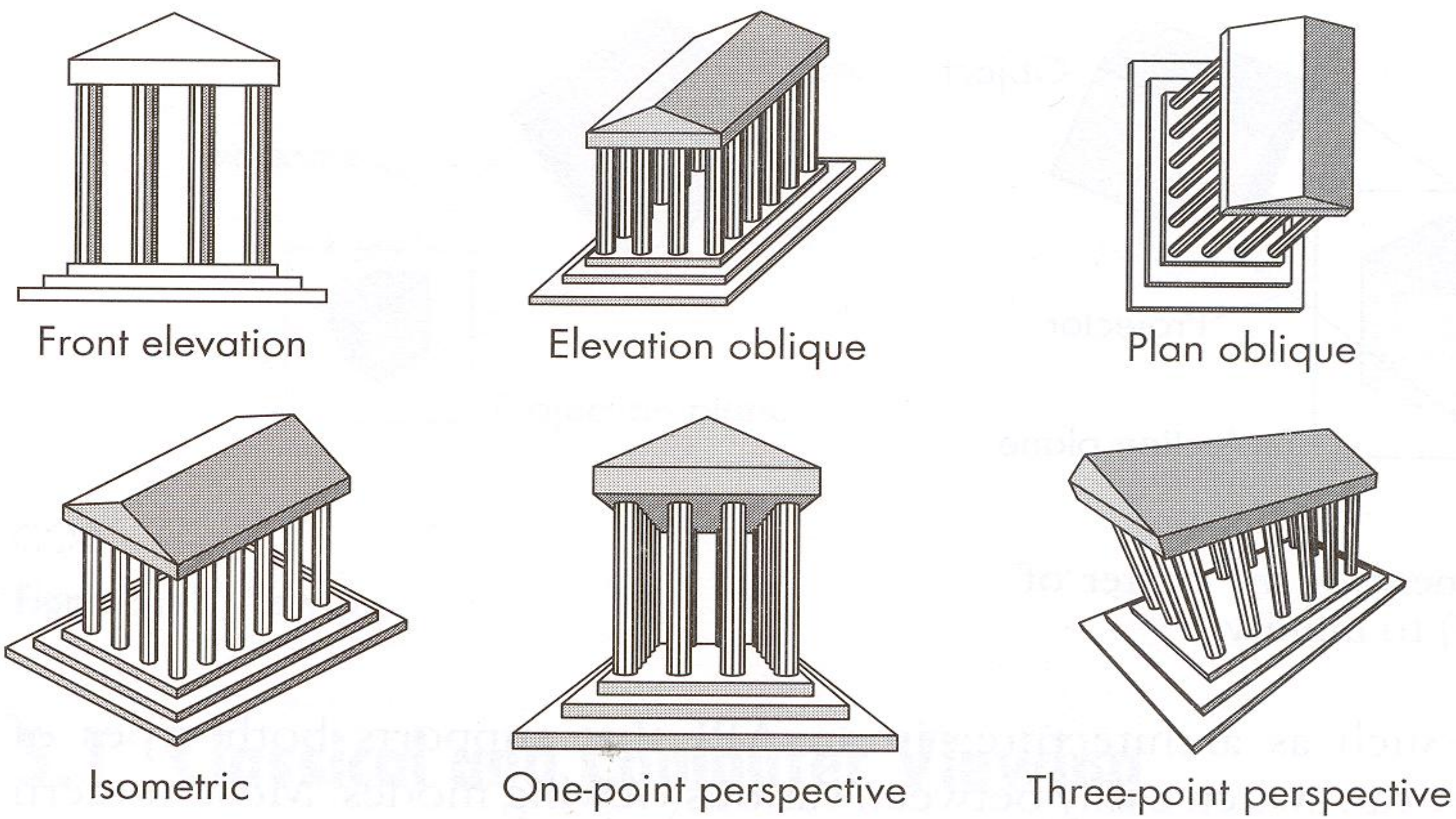
- Perspective projection
  - \+ Size varies inversely with distance - looks realistic
  - – Distance and angles are not (in general) preserved
  - – Parallel lines do not (in general) remain parallel



- Parallel projection
  - \+ Good for exact measurements
  - \+ Parallel lines remain parallel
  - – Angles are not (in general) preserved
  - – Less realistic looking



Classical Projections



A 3D Scene

Notice the presence of the camera, the projection plane, and the world coordinate axes

Viewing transformations define how to acquire the image on the projection plane

Q1: Using the origin as the centre of projection, derive the perspective transformation onto the plane passing through the point $R_0(x_0,y_0,z_0)$ and having normal vector $N=n_1 i+n_2 j+n_3 k$

A: $P(x,y,z)$ is projected onto $P'(x',y',z')$

$x'=\alpha x$, $y'= \alpha y$, $z'= \alpha z$

$n_1x'+n_2y'+n_3z'=d$ (where $d=n_1x_0+n_2y_0+n_3z_0$)

$\alpha=d/(n_1x+n_2y+n_3z)$

$$Per_{N,R0}= \begin{pmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & d & 0 \\ n_1 & n_2 & n_3 & 0 \end{pmatrix}$$

Q2: Find the perspective projection onto the view plane $z=d$ where the centre of projection is the origin(0,0,0)

THANKS!