CBCT: CS 535

Introduction to *Scientific Computing*

Lecture 4

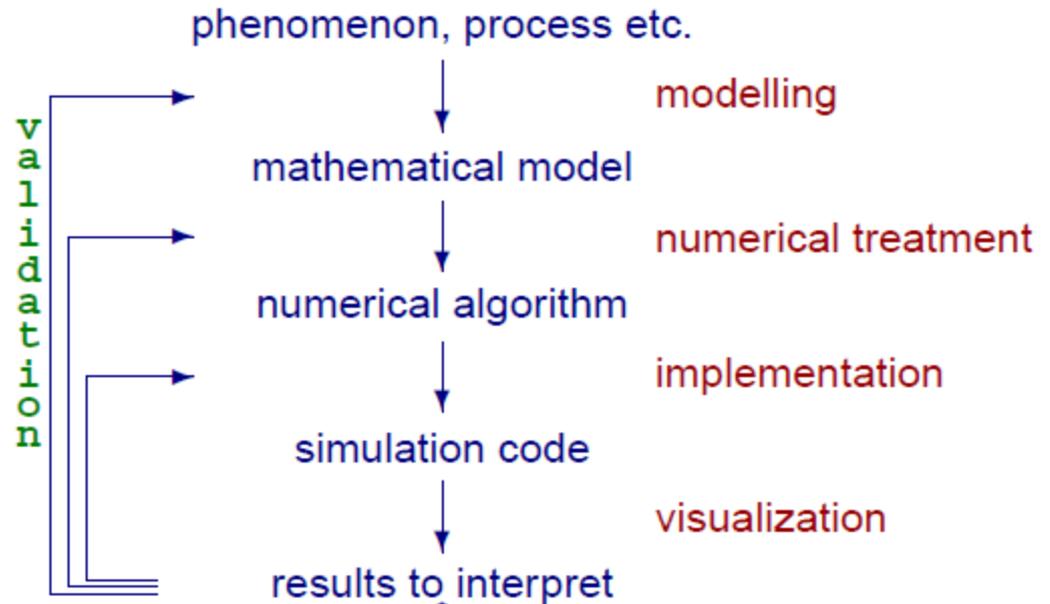Zubin Bhuyan

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Workflow of Scientific Computing

- Problem Identification
  - Find scientifically interesting problems
- Modeling
  - Continuous Representations
  - Discrete Representations
- Simulation
  - Solving equations to solve the system
- Analysis
  - Further processing on results to find meaningful structure or features of interest
- Visualization
  - Creating meaningful representational images of results

phenomenon, process etc.

modelling

mathematical model

numerical treatment

numerical algorithm

implementation

simulation code

visualization

results to interpret

validation

An Example

Consider a system with two parts:
rabbits and wolves

rabbits   eat grass,
        reproduce,
        get eaten

wolves   eat rabbits,
        reproduce,
        die

An Example

- Modelling:
    - A model is a real world process or phenomenon

Assumptions
- Reproduction rates are proportional to food supply and population
- Wolves are the only cause of death among rabbits
- Rabbits are the only source of food for wolves
- Rabbits have an unlimited food supply
- The more rabbits or wolves, the more rabbits get eaten

An Example

- Modelling:
  - A model is represented by mathematical formalism.

Birth and Death Rates

rabbits born per day $=$ rabbit birth rate $\times$ number of rabbits

rabbits eaten per day $=$ number of rabbits $\times$ number of wolves $\times$ chance of getting caught

wolves born per day $=$ wolf birth rate $\times$ number of rabbits $\times$ number of wolves

wolves dying per day $=$ wolf death rate $\times$ number of wolves

An Example

Birth and Death Rates

rabbits born per day = rabbit birth rate × number of rabbits

rabbits eaten per day = number of rabbits × number of wolves × chance of getting caught

wolves born per day = wolf birth rate × number of rabbits × number of wolves

wolves dying per day = wolf death rate × number of wolves

Variables

α = rabbit birth rate

β = chance of getting caught

δ = wolf birth rate

γ = wolf death rate

x = number of rabbits

y = number of wolves

An Example

Birth and Death Rates

rabbits born per day = rabbit birth rate × number of rabbits

rabbits eaten per day = number of rabbits × number of wolves × chance of getting caught

wolves born per day = wolf birth rate × number of rabbits × number of wolves

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Variables

α = rabbit birth rate

β = chance of getting caught

δ = wolf birth rate

γ = wolf death rate

x = number of rabbits

y = number of wolves

Birth and Death Rates

rabbits born per day $= \alpha x$

rabbits eaten per day $= \beta xy$

wolves born per day $= \delta xy$

wolves dying per day $= \gamma y$

An Example

Birth and Death Rates

rabbits born per day $=$ rabbit birth rate $\times$ number of rabbits

rabbits eaten per day $=$ number of rabbits $\times$ number of wolves $\times$ chance of getting caught

wolves born per day $=$ wolf birth rate $\times$ number of rabbits $\times$ number of wolves

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Variables

$\alpha =$ rabbit birth rate

$\beta =$ chance of getting caught

$\delta =$ wolf birth rate

$\gamma =$ wolf death rate

$x =$ number of rabbits

$y =$ number of wolves

Birth and Death Rates

rabbits born per day $= \alpha x$

rabbits eaten per day $= \beta xy$

wolves born per day $= \delta xy$

wolves dying per day $= \gamma y$

Change per Day

$\Delta_{animals} =$ change in the number of animals $=$ births $-$ deaths

$\Delta t = 1$day

$\frac{\Delta_{animals}}{\Delta t} =$ change in the number of animals each day

$\frac{\Delta_{rabbits}}{\Delta t} = \alpha x - \beta xy$

$\frac{\Delta_{wolves}}{\Delta t} = \delta xy - \gamma y$

An Example

- And voilà…..

Lotka-Volterra Equations

- $\frac{dx}{dt} = \alpha x - \beta xy$
- $\frac{dy}{dt} = \delta xy - \gamma y$

First order, non-linear, coupled differential equations.

Thanks!

*Several slides and images have been taken from presentations of S. I. Singh, CSE, TU