CBCT: CS 535

Introduction to *Scientific Computing*

Lecture 15

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Roots??

- If f(x) is a given function, the value of x for which f(x) = 0, is called a **root of the function.**
  - Simple Roots
  - Multiple Roots

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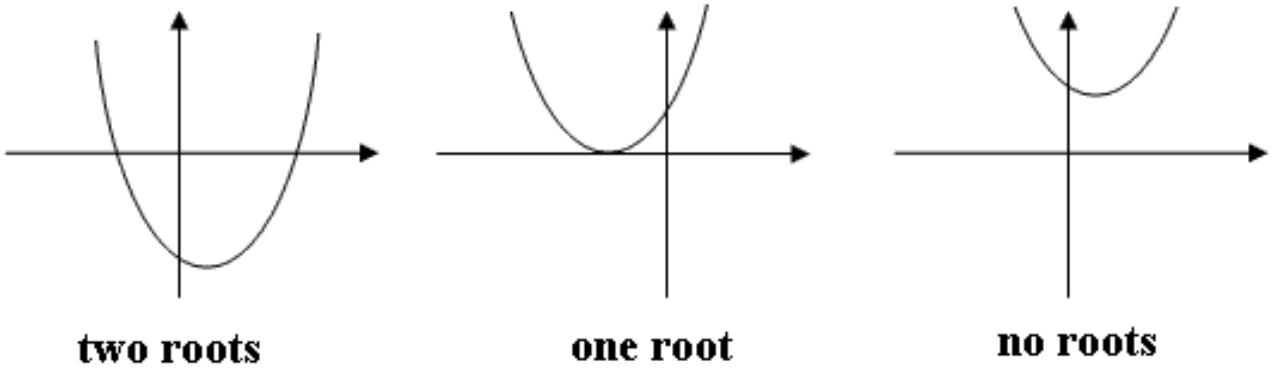
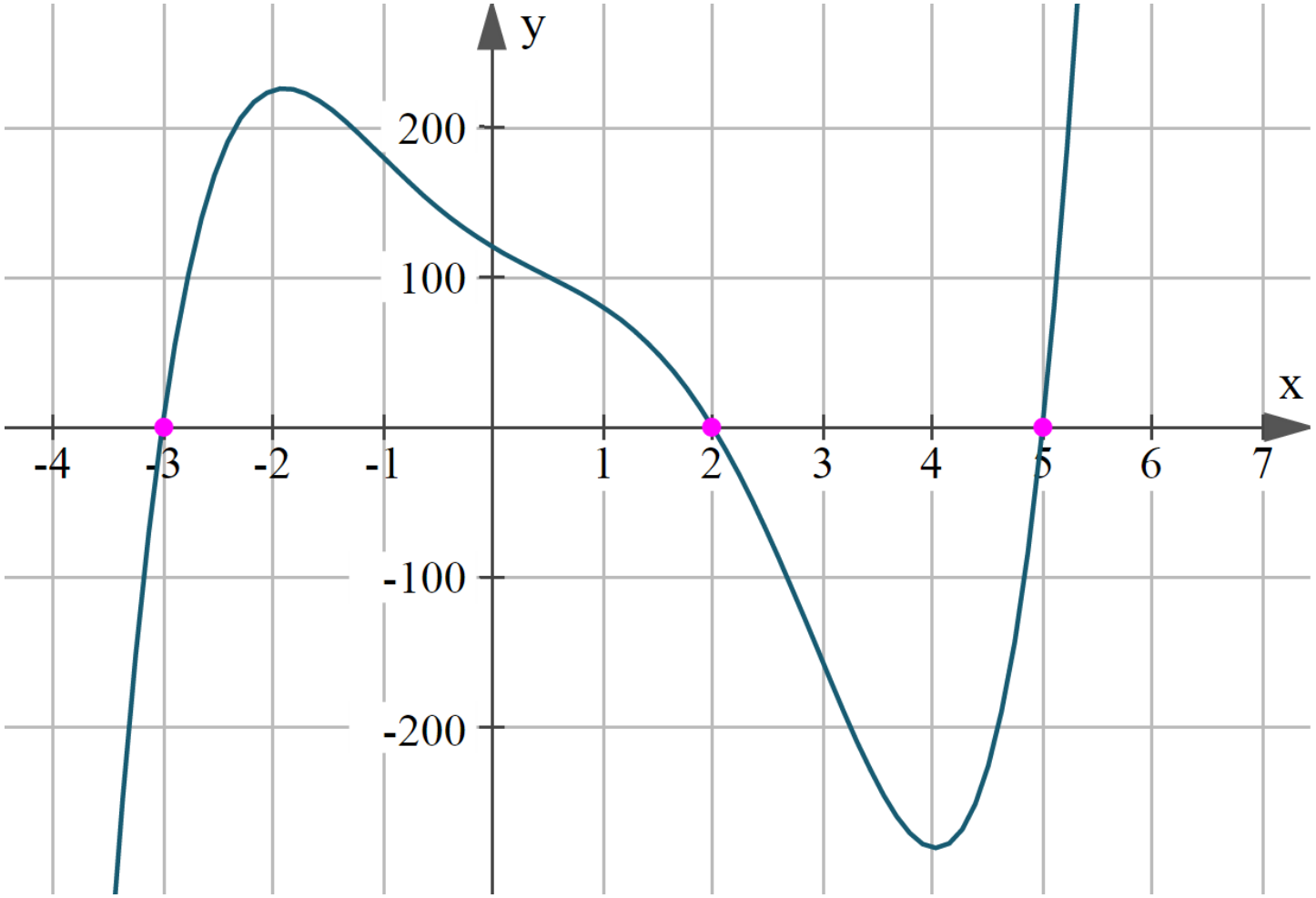


Quadratic Roots

Roots



Bisection Method

- The Bisection method is a numerical method for estimating the roots of a polynomial f(x).
- Always converge, but SLOW!

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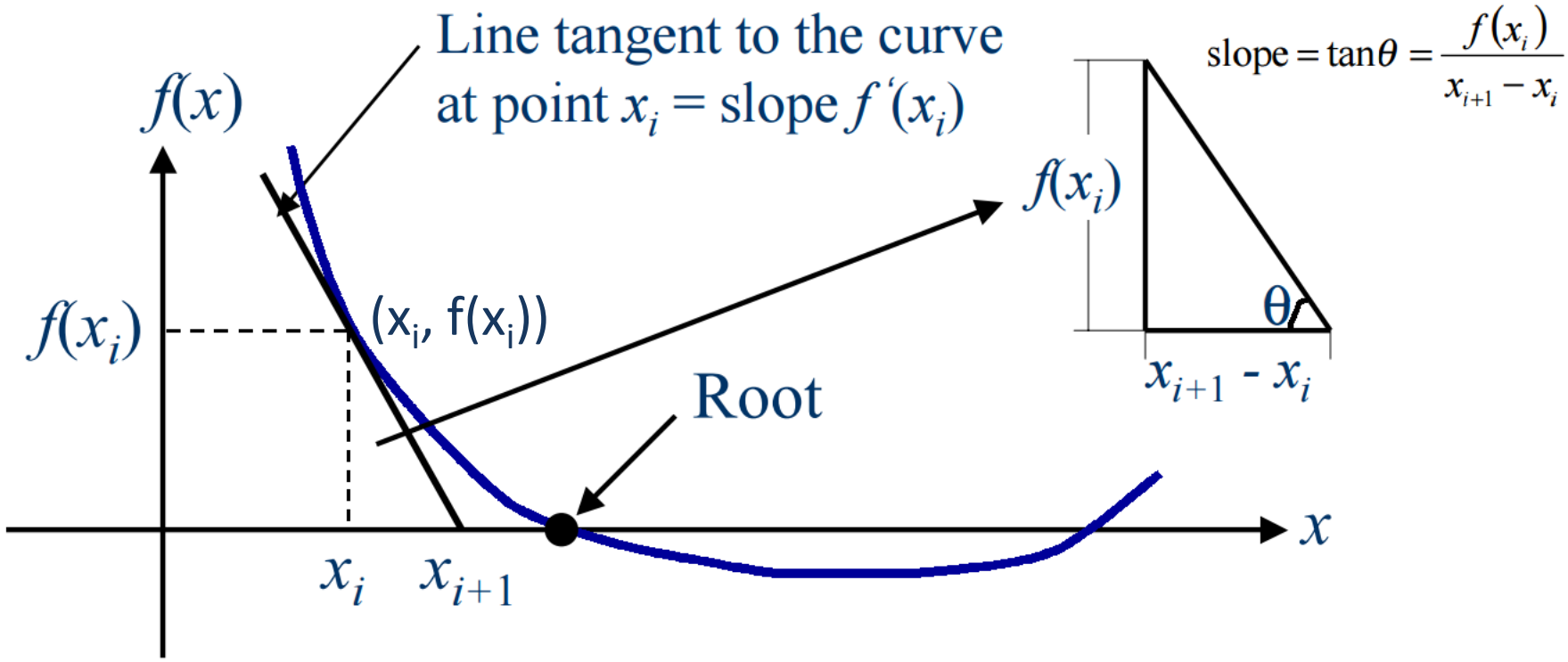
- Newton-Raphson is a faster method!

Newton-Raphson Method

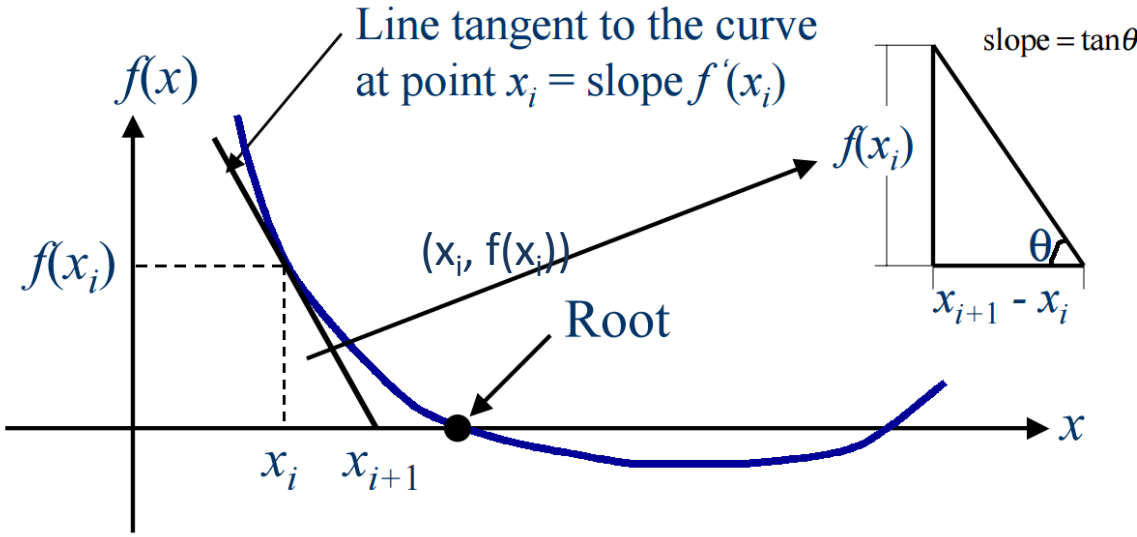
- Used for solving equations, $f(x) = 0$
  - **Find the root** of a non-linear equation

Newton-Raphson Method



Newton-Raphson Method

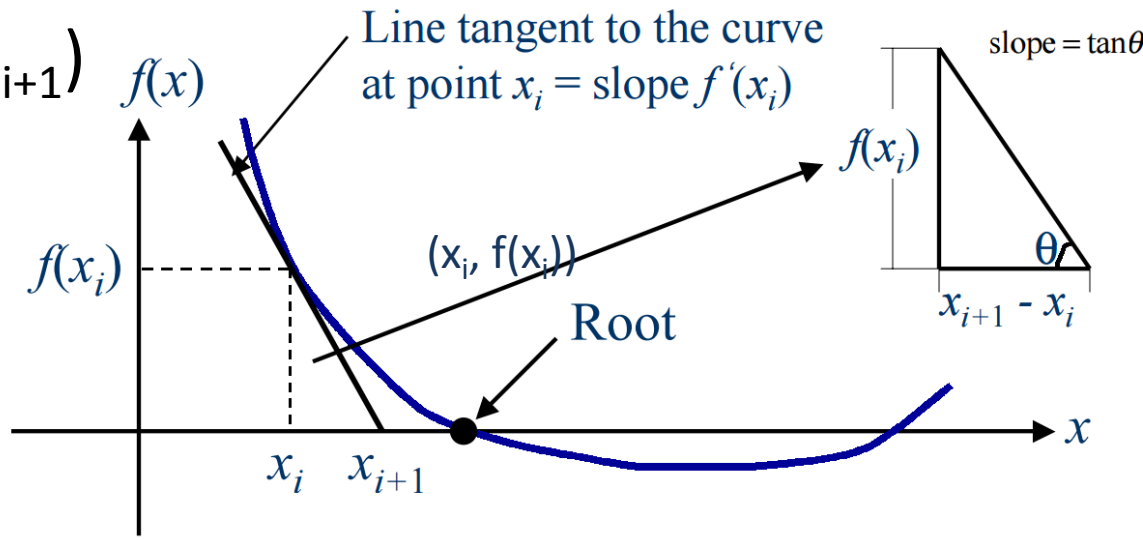
- $f'(x_i) = \tan(\phi)$



Newton-Raphson Method

- $f'(x_i) = \tan(\phi)$

  $= (f(x_i)-0)/(x_i - x_{i+1})$



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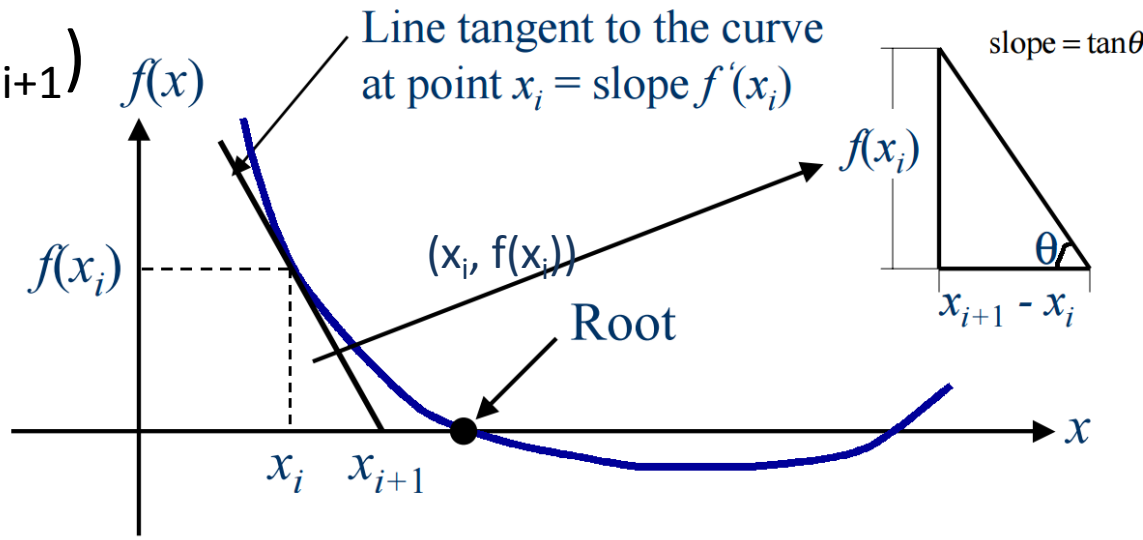

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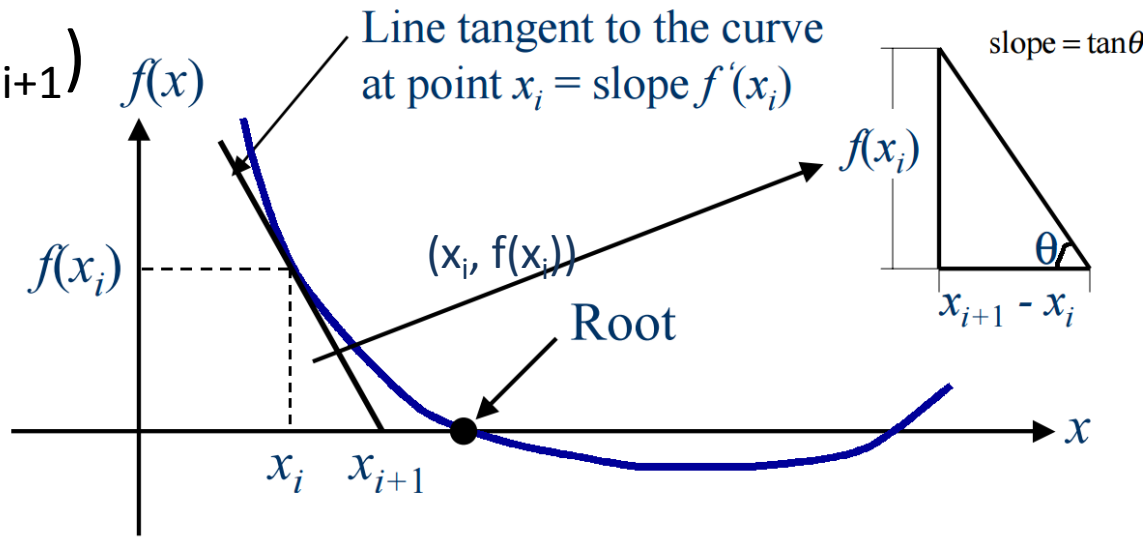
$(x_i - x_{i+1}) = f(x_i)/ f'(x_i)$

$x_{i+1} = x_i - f(x_i)/ f'(x_i)$



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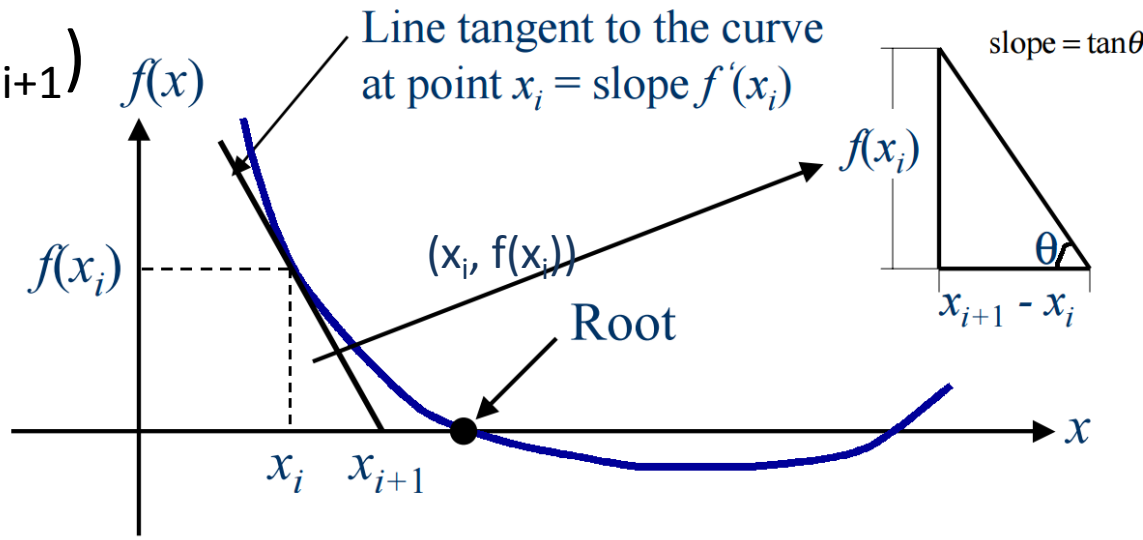

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Newton-Raphson Method

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Newton-Raphson Method

- NewtonRaphson:
  1. Calculate f’(x)
  2. Choose an initial guess, x_0
  3. $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$
  4. Find relative approximate value
     - $\epsilon_a = \left|\frac{x_1 - x_0}{x_1}\right| \times 100$
     - If $\epsilon_a < \epsilon_S$, ϵ_S = some prespecified value

       $x_0 = x_1$, Go to 3

Newton-Raphson Method

- Example: Find the root of

  $f(x) = x^2 - 2x - 2 = 0$

  $f'(x) = 2x - 2 = 0$

  Initial guess, $x_0 = 3$

Newton-Raphson Method

- Example:

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x_0			
3			

Newton-Raphson Method

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  $f'(x) = 2x - 2 = 0$

x_0	f(x)	f'(x)	$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$
3			

Newton-Raphson Method

- Example:

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  $f'(x) = 2x - 2 = 0$

x_0	f(x)	f'(x)	$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$
3	1	4	3-1/4 = 2.75

Newton-Raphson Method

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  $f'(x) = 2x - 2 = 0$

x_0	f(x)	f'(x)	$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$
3	1	4	3-1/4 = 2.75
2.75	0.0625	3.5	2.73214

Newton-Raphson Method

Python program demo.

Thanks!